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HYDROSTATICS

CHAPTER I

Density and Specific Gravity

HYDROSTATICS is the study of forces in fluids at rest and of the forces on solids in contact with fluids.

Three States of Matter

Solids have SIZE and SHAPE and offer permanent resistance to forces which tend to change these properties.

They have rigidity.

Fluids assume the form of the containing vessel and may be further subdivided into *liquids* and *gases*.

Liquids have size or volume and offer strong resistances to forces which tend to change this property ; that is they are only slightly compressible.

Gases fill an enclosure completely and offer only slight resistance to forces which tend to change their volume ; that is they are compressible.

N.B. Border line cases are : elastic, which can be stretched but which within limits returns to its original shape when the stretching force is removed ; pitch, which will flow under the action of quite small forces, in time ; jelly, which can be made of varying degrees of consistency.

Variable and Specific Properties

Examples of variable properties are : colour, volume, weight, shape.

Specific properties are characteristic of the actual substance of which a body is made, and are important in identifying a substance or in estimating its degree of purity when its identity is known.

Such a specific property is DENSITY.

Density is defined as $\frac{\text{Weight}}{\text{Volume}}$ and is expressed in suitable units such as grams per cubic centimetre, pounds per cubic foot, etc.

For example the density of water is 1 gm. per c.c., or 62·3 lb. per cu. ft.

Strictly density $= \frac{\text{mass}}{\text{volume}}$ for mass is the actual quantity of material whereas its weight represents the force with which it is attracted by the earth. This will depend on its nearness to the centre of the earth.

The Measurement of Density

(a) *Regular solids*

The solid is weighed.

Appropriate measurements are made so that the volume can be calculated. Thus volume of cylinder $= \pi r^2 h$; volume of sphere $= \frac{4}{3}\pi r^3$.

Lengths and diameters may be accurately measured to $\frac{1}{100}$ cm. or $\frac{1}{100}$ inch by calipers fitted with a vernier scale. To measure to $\frac{1}{1000}$ cm. or $\frac{1}{1000}$ inch a micrometer screw gauge is used.

(b) *Irregular solids*

The solid is weighed.

Its volume may be found by measuring the volume which it displaces when immersed in water or in any liquid in which it is insoluble.

i. This may be performed by observing the rise of the level of liquid in a graduated vessel when the solid is carefully lowered into the liquid.

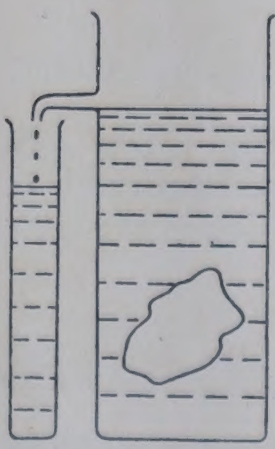

FIG. 1

ii. If the solid is bulky the liquid should be allowed to overflow into a *narrow* graduated cylinder where its volume can be measured more accurately. See fig. 1.

If the solid floats in the liquid, a sinker is tied to it and their combined volume measured. A separate observation will give the volume of the sinker alone.

(c) *Liquids*

A measured volume of the liquid from a burette is run into an accurately weighed beaker.

This is then reweighed to give the weight of the liquid.

N.B. Since the volume of a given weight of substance varies with temperature, its density will also depend on the temperature.

(d) *Air*

A round bottom flask is fitted with a rubber cork, a small piece of glass tubing and a piece of thick walled rubber tubing to which a screw clip is attached.

The flask is weighed when full of air, at atmospheric temperature and pressure, with the clip open.

It is then attached to a filter pump (or other air pump) for two or three minutes. The clip is screwed up tight and the weight of air sucked out is found by reweighing the whole.

The clip is unscrewed under water and the volume of water sucked in may be either measured or weighed to give the volume of air which had been previously removed.

(e) *Other Gases*

A dry density flask, fig. 2, is weighed full of air and again full of the gas, at atmospheric temperature and pressure.

The volume of the flask is obtained later by filling it with water.

Assuming the density of air, the weight of the air in the known volume of flask can be calculated. Hence the weight of the " empty " flask is determined, and so the weight of a known volume of the gas is obtained.

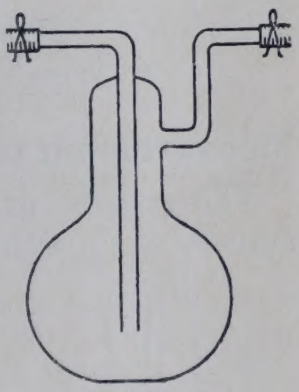

FIG. 2.

N.B. The value obtained for the density of a gas will depend very much on the temperature and on the pressure.

Specific Gravity

For solids and liquids, water is chosen as a standard substance and relative density or **specific gravity** is defined as $\frac{\text{Weight of any volume of substance}}{\text{Weight of equal volume of water.}}$

Thus specific gravity is a *ratio* and *no units* are required.

It should be noticed that :—

$$\text{Specific gravity of substance} = \frac{\text{Wt. of 1 c.c. of substance in gm.}}{\text{Wt of 1 c.c. of water in gm.}} = \frac{\text{Density in gm. per c.c.}}{1}$$

or specific gravity of substance

$$= \frac{\text{Wt. of 1 cu. ft. of substance in lb.}}{\text{Wt. of 1 cu. ft. of water in lb.}} = \frac{\text{Density in lb. per cu. ft.}}{62{\cdot}3}$$

Hence specific gravity and density are the same *numerically* if the density is expressed in gm. per c.c. ; but not otherwise.

For gases, Hydrogen, the lightest gas, is chosen as the standard substance and the specific gravity or **vapour density** of a gaseous substance is then

$$\frac{\text{Weight of any volume of the gas}}{\text{Weight of equal volume of Hydrogen.}}$$

Measurement of Specific Gravity.

The specific gravity can obviously be calculated if the density of the substance and the density of water are known. Specific gravities are, however, usually measured directly.

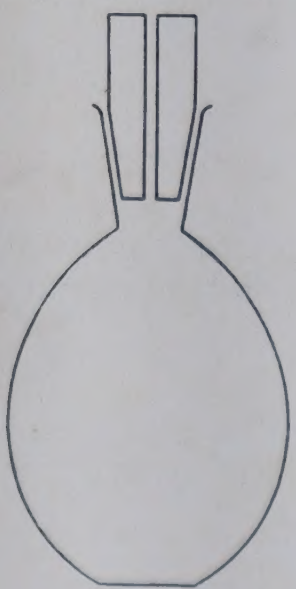

FIG. 3.

Liquids

A clean dry specific gravity bottle, fig. 3, is weighed empty — W_1

It is then exactly filled with liquid, the outside wiped carefully, and weighed again.—W_2

It is washed out with pure distilled water and finally filled with distilled water and weighed again—W_3.

Note that the shape of the ground glass stopper with its capillary ensures that exactly equal volumes of liquid and of water are taken.

Hence $$\text{S.G.} = \frac{W_2 - W_1}{W_3 - W_1}$$

Solids in small state of division

e.g. Lead shot, sand, powdered glass.

The method is illustrated by the following example :

Weigh a clean dry S.G. bottle . . . 15·45 gm.
Introduce powdered solid until the bottle is about one-third full and weigh . . . 86·71 gm.
Fill up with water, taking care to shake out air bubbles, and weigh . . . . . 112·31 gm.
Empty the bottle, refill with water only and weigh . . . . . 66·50 gm.

$\therefore$ Weight of solid = 86·71 − 15·45 = 71·26 gm.

Also weight of water displaced from the bottle by the solid

= (66·50 + 71·26) − 112·31 = 25·45 gm.

$$\therefore \text{S.G.} = \frac{\text{Wt. of solid}}{\text{Wt. of equal volume of water}} = \frac{71{\cdot}26}{25{\cdot}45} = \mathbf{2{\cdot}80.}$$

Solids Soluble in Water

e.g. powdered sugar, salt, etc.

The same method may be applied provided that a liquid such as turpentine, paraffin or benzene can be found in which the solid does not dissolve. The specific gravity of this liquid must be determined, unless it is already known.

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Example.

Wt. of powdered sugar = 25·40 gm.
Wt. of specific gravity bottle filled with turpentine = 56·96 gm.
$\therefore$ Weight together = 82·36 gm.

Wt. of bottle, with sugar inside, otherwise filled with turpentine = 67·82 gm.

$$\therefore \frac{\text{Wt. of sugar}}{\text{Wt. of equal volume of turpentine}} = \frac{25{\cdot}40}{82{\cdot}36 - 67{\cdot}82} = \frac{25{\cdot}40}{14{\cdot}54}$$

S.G. of turpentine = 0·87 (as given or determined).

$\therefore$ S.G. of sugar

$$= \frac{\text{Wt. of sugar}}{\text{Wt. of equal volume of turpentine}} \times \frac{\text{Wt. of this volume of turpentine}}{\text{Wt. of equal volume of water}}$$

$$= \frac{25{\cdot}40}{14{\cdot}54} \times {\cdot}87 = \mathbf{1{\cdot}52.}$$

Other methods for measuring specific gravities of solids and liquids are available depending on Archimedes' Principle (see Chapter III).

The Specific Gravities of Mixtures

Note that the *weight* of a mixture is always equal to the sum of the *weights* of its constituents. This is only true of *volumes* if no expansion or contraction takes place when the constituents are mixed.

Examples

(1) 12 c.c. of a liquid of sp. gr. 1·25 are mixed with 8 c.c. of another liquid of sp. gr. 0·75. Calculate the sp. gr. of the mixture.

12 c.c. of first liquid weigh $12 \times 1{\cdot}25 = 15$ gm.
8 c.c. of second liquid weigh $8 \times 0{\cdot}75 = 6$ gm.

$\therefore$ 20 c.c. of mixture weigh 21 gm.

$\therefore$ 1 c.c. of mixture weighs $\frac{21}{20} = 1{\cdot}05$ gm.

$\therefore$ Sp. gr. of mixture = **1·05.**

(2) A solder of tin and lead has a specific gravity of 9·4. What *weight* of lead (sp. gr. 11·4) is alloyed with 1 gm. of tin (sp. gr. 7·3) in this alloy?

Let x gm. = weight of lead required.

Sp. gr. of lead = 11·4. ∴ 1 c.c. of lead weighs 11·4 × wt. of 1 c.c. of water = 11·4 gm.

∴ Volume of lead weighing

$$x \text{ gm.} = \frac{x}{11{\cdot}4} \text{ c.c.}$$

Similarly volume of

$$1 \text{ gm. of tin} = \frac{1}{7{\cdot}3} \text{ c.c.}$$

∴ Vol. of mixture $= \frac{x}{11{\cdot}4} + \frac{1}{7{\cdot}3}$ c.c.

Also weight of mixture = $1 + x$ gm.

Hence density of mixture

$$= \frac{1 + x}{\frac{x}{11{\cdot}4} + \frac{1}{7{\cdot}3}} \text{ gm. per c.c.} = 9{\cdot}4 \text{ gm. per c.c.}$$

$$\therefore (1 + x) \times 11{\cdot}4 \times 7{\cdot}3 = (7{\cdot}3x + 11{\cdot}4) \times 9{\cdot}4.$$

$$\therefore 83{\cdot}22 + 83{\cdot}22\,x = 68{\cdot}62x + 107{\cdot}16$$

$$\therefore \quad 14{\cdot}6\,x \quad = 23{\cdot}94$$

$$\therefore \quad x \quad = \mathbf{1{\cdot}64 \text{ gm.}}$$

(3) A liquid of specific gravity 1·8 is mixed with water in the proportion one of liquid to four of water by volume.

If the mixture occupies 90 per cent. of the volume of its components what is the specific gravity of the mixture?

1 vol. of liquid weighs 1·8 × wt. of 1 vol. of water

4 vols. of water weigh 4 × wt. of 1 vol. of water.

The volume of the mixture will be $\frac{90}{100} \times 5 = 4{\cdot}5$ vols.

∴ 4·5 vols. of mixture weigh 5·8 × wt. of 1 vol. of water.

∴ 1 vol. of mixture weighs $\frac{5{\cdot}8}{4{\cdot}5}$ × wt. of 1 vol. of water.

$$\therefore \text{ Sp. gr. of mixture} = \frac{5{\cdot}8}{4{\cdot}5} = \mathbf{1{\cdot}29.}$$

(4) A specimen of milk which has been diluted with water has a specific gravity of 1·025. If the specific gravity of the pure milk was 1·03 calculate how much water was added to a pint of the pure milk.

Let x pints of water be added to 1 pint of pure milk.

1 pint milk (sp. gr. 1·03) weighs 1·03 × wt. of 1 pint of water

x pints water weigh x × wt. of 1 pint of water

$\therefore 1 + x$ pints mixture weigh $(1{\cdot}03 + x)$ × wt. of 1 pint of water

$$\therefore \text{1 pint mixture weighs } \frac{1{\cdot}03 + x}{1 + x} \times \text{wt. of 1 pint of water}$$

$$\therefore \text{Sp. gr. mixture} = \frac{1{\cdot}03 + x}{1 + x} = 1{\cdot}025$$

$$\therefore 1{\cdot}03 + x = 1{\cdot}025 + 1{\cdot}025\,x$$

$$\therefore \cdot 025\,x = \cdot 005$$

$$\therefore x = \frac{1}{5} \text{ pint.}$$

Questions on Chapter I

(1) Explain the physical properties in which (*a*) liquids resemble gases, (*b*) liquids differ from gases. (L.)

(2) Define the *density* of a substance.

Describe, in detail, an experiment by which you can determine the density of mercury. (L.)

(3) A reservoir fitted with an overflow pipe is full of water. A piece of rock of weight 1 cwt. and specific gravity 2·5 falls into it. What weight of water overflows and what volume does it occupy? (Density of water 62·5 lbs. per cu. ft.; 1 pint of water weighs a pound and a quarter.)

(4) Explain how the specific gravity of a liquid can be found by using a specific gravity bottle.

A specific gravity bottle weighs 40 gm. when empty and 240 gm. when filled with water. When 260 gm. of sand is placed in the bottle, which is then filled up with water, the

total weight of the bottle and contents is 400 gm. What is the specific gravity of sand? (N.)

(**5**) A specific gravity bottle weighs 20 gm. when empty and 70 gm. when filled with water. Sand, weighing 20 gm. is put into the empty bottle, which is then filled up with water, the total weight being 82 gm. Calculate the specific gravity of the sand.

When the bottle is filled with dry sand the total weight is 103 gm. What percentage of air by volume is mixed up with the sand? (N.)

(**6**) Distinguish between density and specific gravity. Describe how you would determine the specific gravity of a substance such as salt, which is soluble in water, and show how you would calculate your result. (O.)

(**7**) A closed glass flask weighs 72·59 gm. and it weighs 72·01 gm. when partially evacuated. It is then opened under water and 450 c.c. of water enter the flask.

Calculate the density of air under these conditions.

(**8**) Show that if one part by volume of liquid of specific gravity 1·84 is mixed with four parts of distilled water, the specific gravity of the resulting mixture is approximately 1·17. (Assume no change of volume in mixing.)

Describe, in detail, an experiment you could perform in order to verify this value. (L.)

(**9**) Sulphuric acid of density 1·8 gm/c.c. is to be mixed with water to form a solution of density 1·14 gm./c.c. Assuming that there is no change in volume, find the proportion (*a*) by weight, (*b*) by volume, in which the two liquids must be mixed. (O. & C.)

(**10**) A solder of tin and lead has specific gravity 9·5. Find the weight of tin (S.G. 7·3) alloyed with each gram of lead (S.G. 11·4).

(**11**) 200 gm. of a liquid of specific gravity 0·8 are mixed with 100 c.c. of a liquid of specific gravity 1·2. Calculate the specific gravity of the mixture assuming that its volume is equal to the sum of the volumes of its constituents.

(**12**) Distinguish between the density and the specific gravity of a body. Show how these are represented by the

same number only if the c.g.s. system of units be employed.

A volume of 100 c.c. of water is mixed with an equal volume of alcohol, of specific gravity 0·8. At the same temperature as before mixing the specific gravity of the mixture is found to be 0·92.

Calculate the percentage volume contraction which has taken place. (N.)

N.B. For further examples on density and specific gravity see Chapter III.

CHAPTER II

Pressure

Pressure is defined as $\frac{\text{Thrust or Weight}}{\text{Area}}$ and is expressed in grams per square centimetre or in pounds per square inch, etc.

In the case of a uniform cube of solid resting exactly on a flat horizontal chessboard, the weight supported by each square of the chessboard will be the same, and the pressure is said to be uniform.

When the thrust on an area is not uniformly distributed, e.g., a heap of sand on the chessboard, the idea of pressure at a point must be considered.

The pressure at a point is the thrust on a very small imaginary area surrounding that point divided by the area.

In order to apply this idea certain important propositions or theorems must be understood :

(1) **The pressure at a point in a fluid, at rest, has the same value in all directions.**

Experimental illustration

Two similar tall glass jars are filled with water and two narrow glass tubes containing mercury, as shown in figs. 4 (*a*) and 4 (*b*), are placed in them.

The pressure of the water causes the mercury to rise in the left hand limb of each narrow tube.

The specific gravity of mercury is 13·6 so that this rise is small but it is the same in the two cases showing that the pressure at a certain depth is the same in a vertical as it is in any other direction.

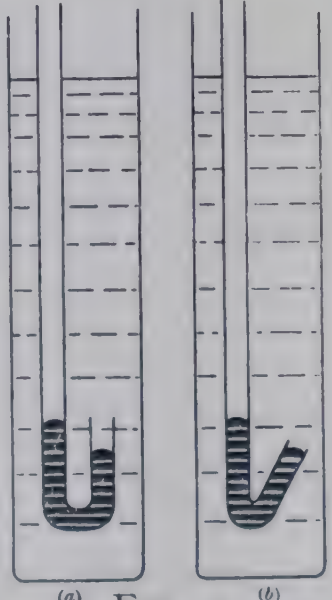

FIG. 4.

(2) **The pressure at a point in a *liquid at rest* is proportional to the depth of the point below the surface and to the density of the liquid.**

Assuming theorem (1) it is sufficient to calculate the pressure in one direction only, viz., the vertical, which is the easiest.

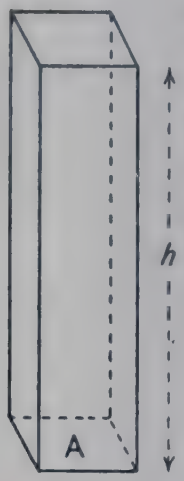

FIG. 5.

A small horizontal area A sq. cm. is described around a point which is at a depth h cm. in a liquid of density d gm. per c.c.

The volume of liquid vertically above this small area is $A \times h$ c.c.

The weight of liquid vertically above this small area is $A \times h \times d$ gm.

Hence the pressure at the point is

$$\frac{A \times h \times d}{A} \text{ gm. per sq. cm.}$$

$$= h \times d \text{ gm. per sq. cm.}$$

Thus the pressure is directly proportional to h and to d.

In the case of *gases* this will not be true for the density of the gas will differ at different depths, for the density increases with the pressure.

Experimental Illustrations

(*a*) Three exactly similar jets are inserted through corks in holes at different levels in a jar containing water. The curves formed by the water suggest that pressure is greater as depth increases. See fig. 6.

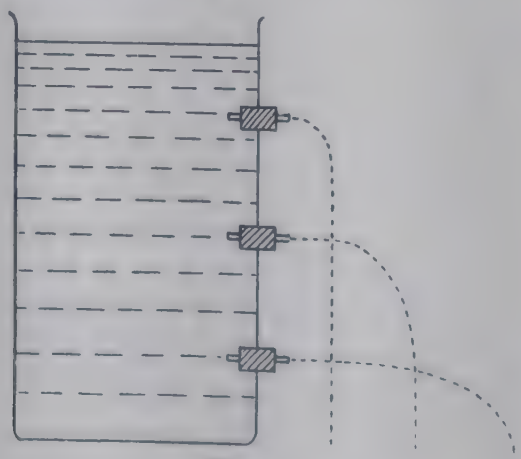
FIG. 6

(*b*) A thin metal disc is held against the bottom of a tube which can be lowered into a jar of water. Below a certain depth the string may be released and yet the disc remains in position. See fig. 7.

As the tube is gradually raised the depth at which the disc just falls can be measured.

This can be repeated with a series of discs of different weights and it can be shown that the depths are proportional to the weights supported.

If another liquid is substituted for the water it can be shown that the depth at which a given plate just falls is *inversely proportional* to the density of the liquid. Thus the pressure at a given depth is *proportional* to the density.

FIG. 7

(*c*) The apparatus of fig. 4 can be used to illustrate the fact that the pressure in any liquid is directly proportional to the depth and the density.

The difference in level of the mercury in the two arms of the tube is measured at various depths in, say, water. The experiment is then repeated using another liquid, of known density, measurements being made at the same depths as before.

(3) **The pressure at any points in the same horizontal line in a fluid at rest is the same, and a liquid finds its own level.**

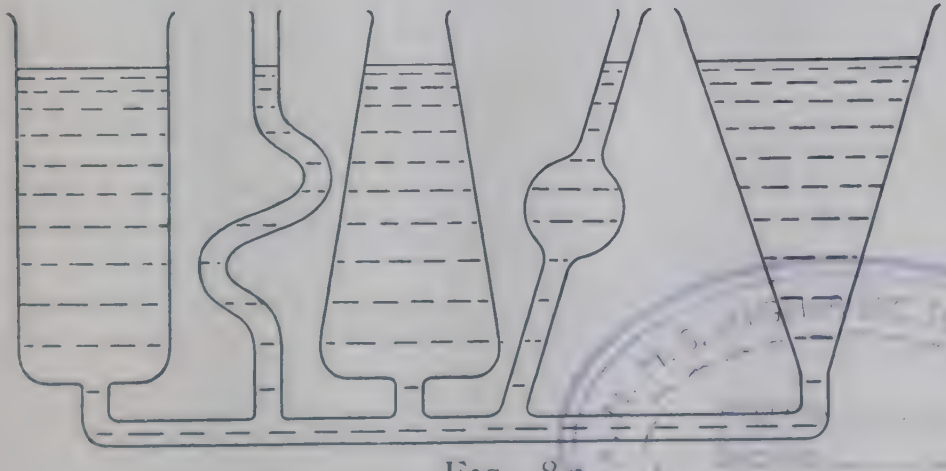

FIG. 8

This is shown in the apparatus fig. 8, well known as Pascal's Vases.

This principle is utilised in artesian wells, domestic water supply, petrol gauges.

(4) **The pressure at a point on a surface immersed in a fluid at rest is at right angles to the surface.**

(If this were not so the fluid would move, but it is at rest.)

The total thrust on any surface immersed in a fluid.

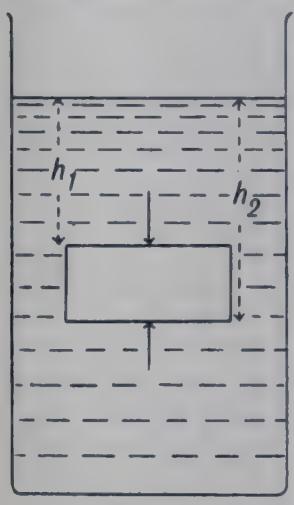

FIG. 9

Fig. 9 shows a rectangular slab of substance, with its faces vertical and horizontal, immersed in a liquid.

It will experience a downward thrust on its upper surface equal to the pressure at depth h_1 multiplied by the area of the surface.

Similarly there is an upward thrust on the lower surface due to the pressure at depth h_2. If the difference between these thrusts is greater than the weight of the substance then the substance will rise and *vice versa*.

This argument will hold at any depth provided that the body is not compressible and that the density of the liquid is not affected appreciably by changes of pressure.

(5) **The total thrust on any plane surface immersed in a fluid at rest is obtained by multiplying the area by the pressure at the "centre of area" of the subject.**

For further information about solids immersed in liquids, see Chapter III.

To measure the pressure of the gas supply.

A small U-tube containing coloured water is connected to the gas supply. Fig. 10.

Pressure of gas supply
= Pressure at A
= Pressure at B (in same horizontal line)
= Pressure due to depth h + Pressure of atmosphere.

In practice, pressures are often expressed in terms of the height of water (or mercury) which they would balance.

In other cases the pressure is measured in pounds weight per square inch (or in gm. wt. per sq. cm.).

To change from one set of units to the other :—

Pressure at any depth in a liquid = Depth × Density (in excess of that on the liquid's surface).

N.B. If the depth is measured in **inches**, *the density must be expressed in lb. per cu.* **inch** ; *etc.*

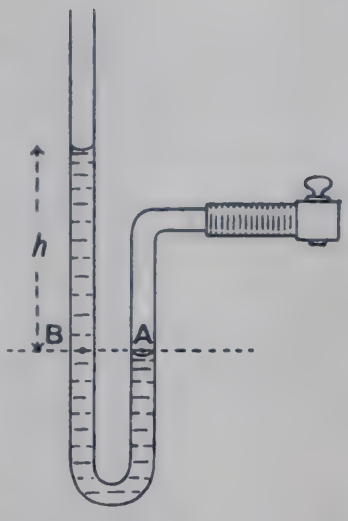

FIG. 10

Thus the pressure due to 50 inches of water

$$= 50 \times \text{density of water in lb. per cu. in.}$$

$$= 50 \times \frac{62{\cdot}3}{1728} \text{lb. per sq. in.}$$

To compare the densities of two liquids

Fig. 11 (*a*) illustrates a U-tube containing two liquids which do not mix.

Fig. 11 (*b*) illustrates a U-tube containing two liquids which would mix but which are separated by some mercury,

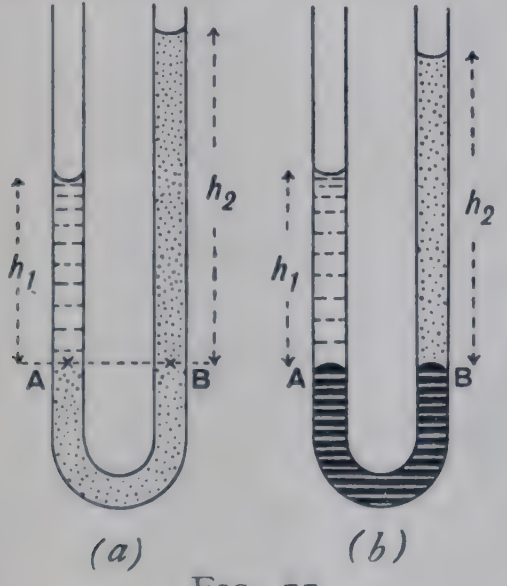

FIG. 11

the quantities of liquids being adjusted to make the mercury in the two limbs at the same level.

Pressures at points A and B are equal.

$\therefore h_1 \times d_1$ gm. per sq. cm. $= h_2 \times d_2$ gm. per sq. cm.

where h_1, h_2 are the depths and d_1, d_2 the densities.

$$\therefore \frac{d_1}{d_2} = \frac{h_2}{h_1}$$

If the liquid in the right hand limb is water then $d_2 = 1$ *gm. per c.c. and* $\frac{h_2}{h_1}$ *then gives the relative density or specific gravity of the other liquid.*

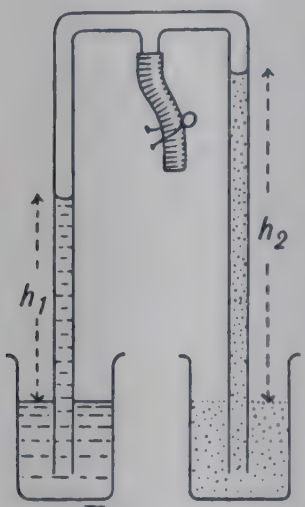

FIG. 12.

N.B. As the pressure depends only on depth it is clear that the limbs of the U-tube need not necessarily be of equal nor of uniform cross sections.

Hare's Apparatus. Fig. 12.

The U-tube is inverted and the pressure at the top is reduced by sucking out some of the air and clipping the rubber tubing.

A series of readings for the ratio of the two heights should be obtained by varying this reduced pressure.

The Siphon. Fig. 13.

Pressure on the outside of the right hand limb of tube = Atmospheric Pressure.

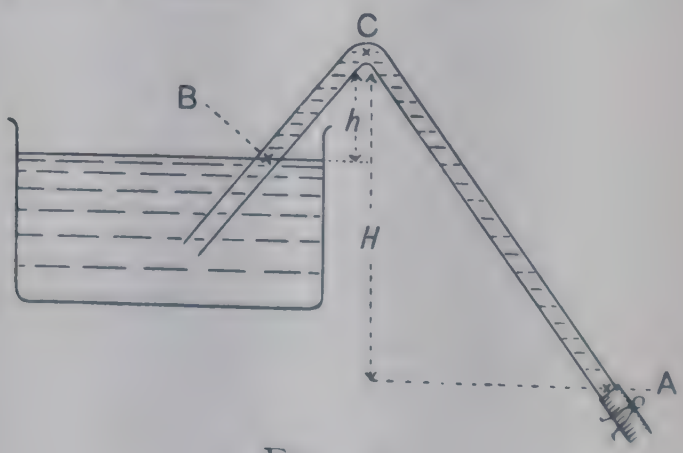

FIG. 13.

Pressure at A just inside clip
= Pressure at C + pressure due to height H of liquid
= (Pressure at B — Pressure due to height h) + pressure due to H

Pressure at B = Atmospheric Pressure.

Hence pressure just inside the clip — pressure just outside = H — h

This will only be true if the siphon tube is completely full of liquid which is at rest.

Hence when the clip is opened liquid will run out and continue to do so, as long as H remains greater than h.

Pressure in Gases

Owing to the comparative lightness of gases the variations of pressure in small columns of gases at rest are quite small and, in experiments on liquid pressure, may be neglected.

The Pressure of the Atmosphere

The fact that the atmosphere exerts a considerable pressure is often shown by using an air pump to remove the air from a thin metal can, which then collapses under the forces due to the external pressure.

The simple barometer. Fig. 14.

A dry glass tube 36 in. long, closed at one end is completely filled with mercury, and the open end closed by the thumb.

It is placed in a bowl of mercury and the thumb removed. The mercury falls, leaving a vacuum in the top of the tube.

Atmospheric Pressure at A
= Pressure at B (in same horizontal line)
= Pressure at a depth H in mercury.

Note that if the tube is sloped the *vertical* distance between the two mercury surfaces will still be H.

Moreover H will remain the same even if the tube changes in cross-section.

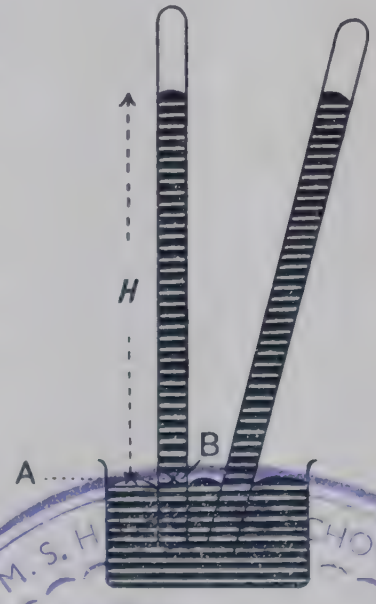

FIG. 14.

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Calculation of atmospheric pressure.

The usual height of the mercury column in a barometer tube is about 76 cm. (about 30 inches).

Hence the pressure is equivalent to a thrust due to 76 c.c. of mercury on 1 sq. cm.

$$\therefore \text{ pressure } = 76 \times 13{\cdot}6 \text{ gm. weight per sq. cm.}$$
$$= 1033{\cdot}6 \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{,,}$$

Pressures are often expressed in "dynes per sq. cm." where the dyne is the so-called "absolute unit of force." This absolute unit is necessary because experiment shows that the gm. wt. has slightly different values at different places on the earth's surface.

At London 1 gm. wt. = 981 dynes.

Hence the above pressure of 1033·6 gm. wt. per sq. cm.
= 1033·6 × 981 dynes per sq. cm.
= 1 million dynes per sq. cm. approx.

In English Units, assuming H to be 30 inches the pressure is equivalent to a thrust due to 30 cubic inches of mercury on 1 sq. inch.

But sp. gr. of mercury = 13·6.

Hence 1 cubic inch weighs $\frac{13{\cdot}6 \times 62{\cdot}3}{1728}$ lb.

Hence pressure $= \frac{13{\cdot}6 \times 62{\cdot}3}{1728} \times 30$ lb. per sq. inch
= **14·7** lb. per sq. in.

Water Barometer

If the mercury in a simple barometer registers 30 inches, the equivalent column of water would be 30 × 13·6 inches = 34 ft.

Pressure and altitude

Pressures at different heights in the air cannot be calculated from the simple formula, depth × density, because the density of a gas varies considerably with pressure and therefore with altitude.

The relation between height and pressure in the atmosphere is a complicated one, but a formula can be obtained

to enable altitudes to be calculated from barometric readings. Aeroplane pilots and mountaineers usually estimate their altitude by means of a special type of barometer.

For small changes of height near sea level the barometer falls $\frac{1}{10}$ in. for 90 ft. increase in altitude.

Fortin's Barometer

When the mercury level, in the simple barometer, changes, small corresponding changes occur in the level of the mercury in the bowl. Hence an attached scale whose zero reading is correct for one level of mercury will be out of adjustment for another.

In *Fortin's Barometer* this trouble is avoided thus :

The mercury in the bowl is contained in a leather bag, which rests on the top of a flat screw. When this screw is turned the surface of the mercury is raised or lowered, until it just touches a fixed ivory pointer. The scale on this barometer records the height of the mercury surface relative to the tip of the ivory pointer.

Usually only the readings from 28 to 31 inches are marked, and a vernier is provided so that very accurate readings can be taken if required.

Aneroid Barometer

An Aneroid Barometer does not contain any liquid.

It consists, essentially, of a metal box, with a thin corrugated top. The box is partially exhausted.

With changes of atmospheric pressure the top rises or falls and these small movements are magnified and transmitted by a series of levers to a pointer. This moves over a dial, or in a " barograph " over a paper scale rotated very slowly by clockwork.

The detailed construction of both an Aneroid and a Fortin Barometer can best be understood by a careful examination of the instruments.

Boyle's Law 1664

The volume of a given mass of gas is inversely proportional to the pressure of the gas, provided that the temperature remains constant.

$$V \propto \frac{1}{P} \text{ or } PV = \text{constant or } P_1V_1 = P_2V_2 = P_3V_3 \text{ etc.}$$

Thus if the volume of a given mass of gas at a pressure of 750 mm. of mercury is 190 c.c., at 760 mm. of mercury it will become V_2

where $760 \times V_2 = 750 \times 190$

Hence $V_2 = \frac{750 \times 190}{760}$ c.c. = **187·5** c.c.

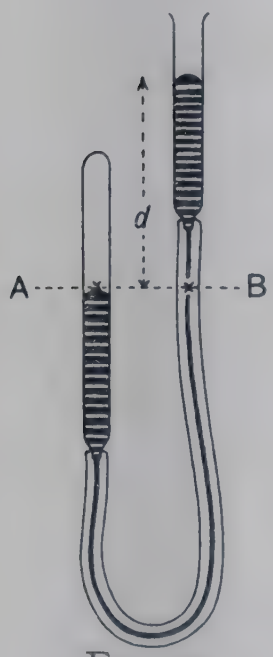

FIG. 15

Experimental Verification

Air is enclosed in the left hand tube of the apparatus shown in fig. 15. The right hand tube is open to the atmosphere and the tubes are joined by thick walled rubber tubing which, together with the lower portions of the tubes, contains mercury.

The left hand tube is of uniform cross-section and the length occupied will be proportional to the volume of the air.

The pressure of the enclosed air
= Pressure at A
= Pressure at B
= Atmospheric Pressure + Pressure due to column of mercury of height d.

The pressure can be varied by raising or lowering the right hand tube and the mercury levels are read on a scale fixed to the apparatus.

Results may be tabulated thus :

Barometer reading = 75 cm.

Length of enclosed air	*Difference of mercury levels*	*Pressure of air*	*Product Pressure × length of air*
21·2 cm.	2·4 c.m.	75 + 2·4 = 77·4 cm.	1641
18·5 ,,	13·8 ,,	75 + 13·8 = 88·8 ,,	1643
23·3 ,,	−4·5 ,,	75 − 4·5 = 70·5 ,,	1643

Boyle's Law is applied in the measurement of pressures by means of a " closed air manometer," fig. 16.

When the tube A is in contact with the atmosphere the pressure in the closed tube is (approx.) 15 lb. per sq. in. l is then say, 6 inches.

If A is connected to a motor tyre and the length l becomes 2 inches, then

$$15 \times 6 = P \times 2$$

Hence P = 45 lb. per sq. in., i.e., 30 lbs. per sq. in. above atmospheric pressure.

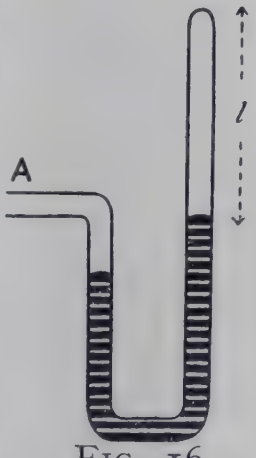

FIG. 16

Typical Examples on Boyle's Law.

1. *A faulty barometer contains some air above the mercury. The length of the mercury when the open end of the tube is just beneath the surface of the mercury in the reservoir is 74 cm. When the tube is depressed so that the volume of the space above the mercury is halved, the length of the mercury column is only 72 cm. Calculate the atmospheric pressure.*

Let P cm. be the atmospheric pressure and let x c.c. be the original volume of air space.

Then original pressure in space above mercury = (P − 74) cm.

Hence $(P - 74)\, x = (P - 72)\, \frac{x}{2}$ by Boyle's Law

$$\therefore \quad P = \mathbf{76\ cm.}$$

2. *A cylindrical diving bell 24 ft. high is lowered into water. Find the depth of the top below the surface of the water when the bell has become half full of water. (Atmospheric pressure is equivalent to 34 ft. of water.)*

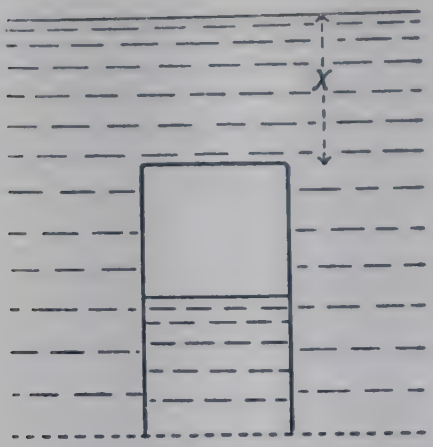

FIG. 17

Let x ft. be the required depth, see fig. 17.

At the surface Pressure = 34 ft. and Volume of enclosed air = V units.

In lowered position Pressure $= 34 + x + \frac{24}{2}$ and Volume of enclosed air $= \frac{V}{2}$ units.

Hence by Boyle's law

$$34 \times V = (34 + x + 12) \times \frac{V}{2}$$

$$\therefore 34 = 23 + \frac{x}{2}$$

$$\therefore x = \mathbf{22\ ft.}$$

3. *The mercury in the open limb of a U-tube is* 5 *cm. above that in the other which encloses* 10 *cm. of air.*

How much mercury must be poured into the open limb to reduce this column of air to 8 *cm.? (Atmospheric Pressure* = 75 *cm.).*

Volume of air is reduced to $\frac{8}{10}$ of its original value.

Hence new pressure must be $\frac{10}{8}$ of its original value, i.e.,

$$\frac{10}{8}(75 + 5) = 100 \text{ cms.}$$

Hence pressure to be provided by mercury

$$= 100 - 75 = 25 \text{ cms.}$$

i.e., Mercury in open limb must be 25 cms. above that in closed limb.

Of the original 5 cm. difference of level, 1 cm. would remain when mercury in closed limb rises 2 cm., since $5 - (2 \times 2) = 1$ cm.

Hence an additional $25 - 1 = 24$ cm. of mercury must be poured in.

Transmission of Fluid Pressure

A Change of *Pressure* occurring at any point in a fluid at rest is transmitted to all other points.

Fig. 18 represents a U-tube with limbs of different cross sections A and *a* sq. cm. respectively. The tube is filled with a fluid and the two ends are closed by tightly fitting but frictionless pistons, carrying scale pans. These just balance each other.

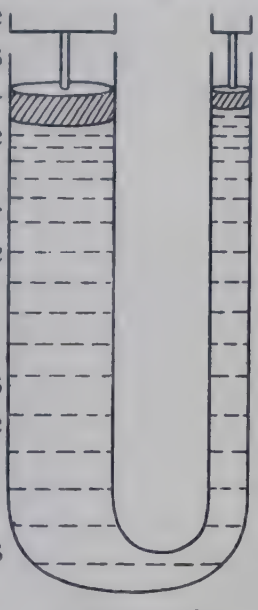

FIG. 18

A weight W gm. is placed on the larger pan and produces an increase of *Pressure* of $\frac{W}{A}$ gm. per sq. cm.

This increase is transmitted to all points and so produces an increased *thrust* on the smaller piston of $\frac{W}{A} \times a$ gm.

The weight necessary to balance this would be *w* gm.

$$\text{where } w = \frac{W}{A} \times a \text{ or } \frac{W}{A} = \frac{w}{a}$$

If practical difficulties could be surmounted a very minute weight could thus balance a very large one.

This principle of the " Hydrostatic Balance " is used in hydraulic machines, lifts, and presses.

The Bramah Press

This is shown in diagrammatic form in Fig. 19.

It will be noticed that the effort required, representing the small weight above, is further diminished by using a handle as a lever.

The escape of water between piston and cylinder is avoided by using cup-shaped leather washers which become tighter as the pressure increases.

$\frac{\text{The Load Lifted}}{\text{The Effort Required}}$ is called the *Mechanical Advantage* of the Press.

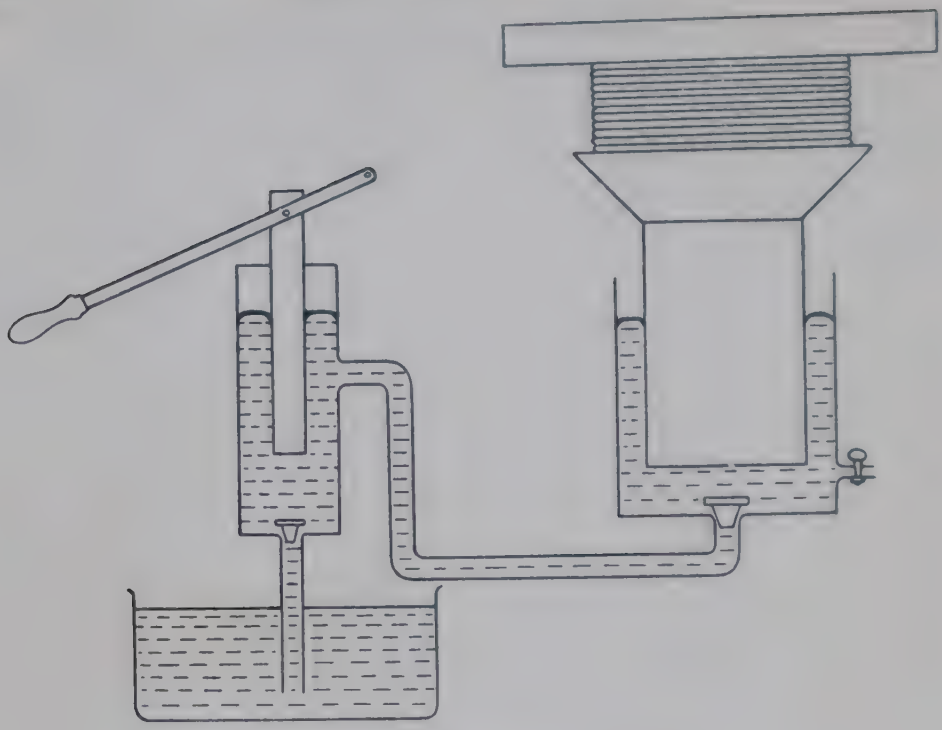

FIG. 19

$\frac{\text{Distance moved by Effort}}{\text{Distance moved by Load}}$ is called the *Velocity Ratio*.

Hence

$$\frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}} = \frac{\text{Load} \times \text{the distance it moves}}{\text{Effort} \times \text{the distance moved by Effort}}$$

$$= \frac{\text{Work done on Load}}{\text{Work done by Effort.}}$$

This ratio $\frac{\text{Work got out}}{\text{Work put in}}$ is called the *Efficiency* of the machine and will always be less than unity.

Pumps

Ordinary Lift Pump. Fig. 20.

The student should make certain that he can describe in detail the action of the pumps shown diagrammatically.

The distance AB must be appreciably less than 34 ft. (the height of the water barometer).

In a very deep well the rod C may be of considerable length in order to keep AB less than about 25 ft. Fig. 20.

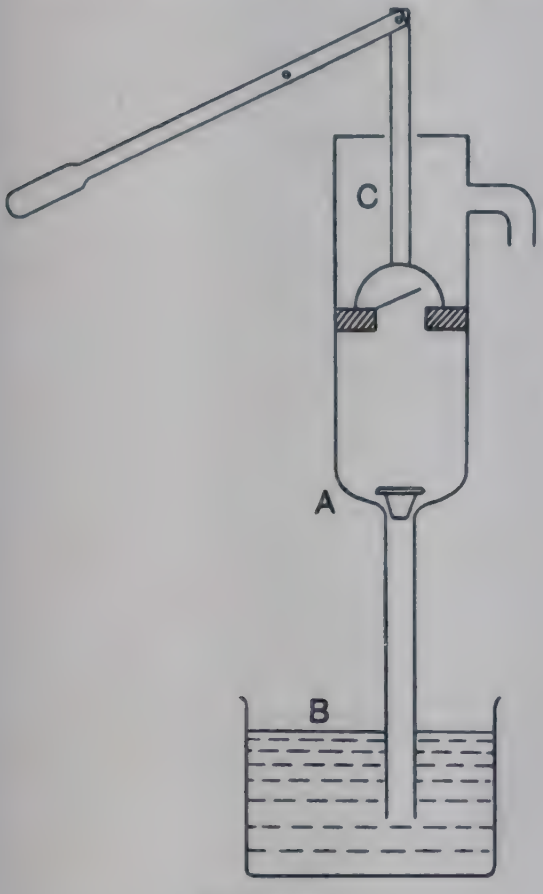

FIG. 20

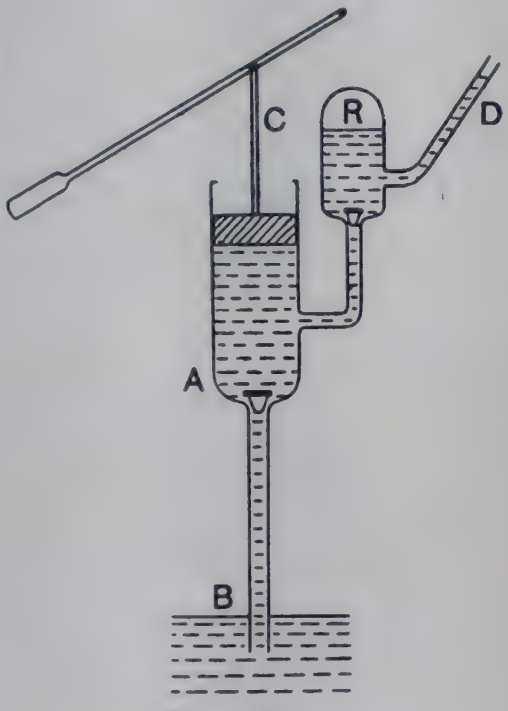

FIG. 21

When this pump is in action the force exerted on the down stroke is small, as the water is lifted during the up stroke. Hence the large force in the rod C is a tension which does not bend the rod.

Force Pump. Fig. 21.

Again AB is rarely greater than 25 ft.

The rod C in this form of pump does most of its work during the down stroke, and the water can be forced to any height up the tube D.

The reservoir R of compressed air helps to give a continuous flow of water.

Air Pumps

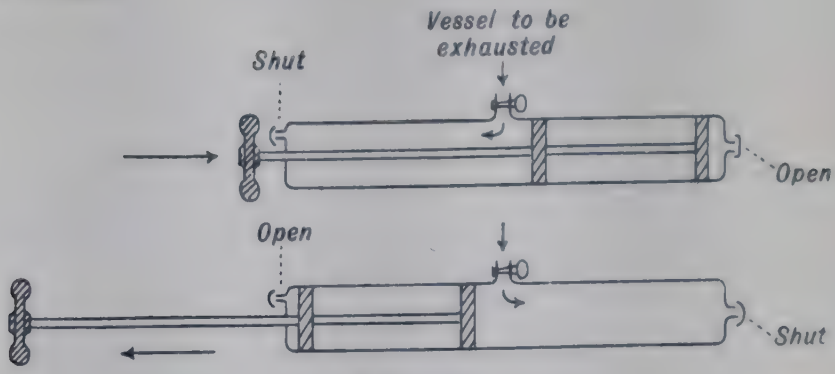

FIG. 22

There are numerous varieties; the ordinary bicycle pump is a compression pump, the Fleuss air-pump is an exhaust pump.

Fig. 22 shows diagrammatically the principle of the double-acting exhaust pump used in many school laboratories.

Questions on Chapter II

(**1**) Define *force* and *pressure*, and state in what units each is measured.

A cubical tank, each edge of which measures 2 ft. internally, is half full of brine, of specific gravity 1·025. Calculate (*a*) the *pressure* at the bottom of the tank, (*b*) the *force* acting on each side, due to the brine.

Find also what addition is required to the answers to (*a*) and (*b*) respectively if allowance is made for the action of atmospheric pressure on the inside surfaces of the tank, the water barometer standing at 33 ft.

(A cub. ft. of water weighs 1,000 oz.) (N.)

(**2**) Calculate the pressure at a depth of half a mile beneath the surface of the ocean, taking the density of sea water as 64 pounds per cubic foot.

Suggest possible means by which investigators who go down to such depths may maintain atmospheric pressure within the spheres in which they descend, while the oxygen content of the air is kept reasonably constant. (A.)

(**3**) What is meant by the *pressure at a point* in a liquid? Describe an experiment to show how the pressure at a

point in a liquid increases with the depth below the surface.

Calculate in dynes per sq. cm. the pressure at a point 100 metres below the surface of the sea on a day when the height of the mercury barometer at sea level is 745 mm. (Specific gravity of sea water = 1·03. Specific gravity of mercury = 13·6.) (O. & C.)

(**4**) State *the laws of liquid pressure.*

A conical vessel is 20 cm. high and the base diameter is also 20 cm. It is filled with water to half its height. Calculate the pressure at any point on the base, and the total thrust on the base.

Calculate the weight of water that would fill the vessel. Compare this weight with the value obtained for the total thrust on the base and comment on the result. (N.)

(**5**) Draw and describe Hare's Apparatus, and explain how you would use it to determine the density of brine. Deduce the formula you use.

Give any *one* other method which you could use to check your result. (N.)

(**6**) Describe in detail how you would compare the density of coal gas with that of air.

The pressure of the gas supply to a town is commonly about "four inches of water." State exactly what is meant by this and express it in pounds per square inch. Give a sketch of the device that you would use to measure the pressure of the gas and indicate how it works. (A.)

(**7**) Water is poured into a glass U-tube of uniform cross-section 1 sq. cm. until each limb (30 cm. high) is half-filled. Explain why the level of water is the same in both limbs.

20 c.c. of paraffin (sp. gr. 0·8) are carefully poured into one limb. What is the final height, above the bottom of the tube, of the surface of separation between the water and paraffin? (N.)

(**8**) Describe and explain the action of a siphon.

On what factors will the volume of water siphoned per minute depend?

Calculate the maximum heights through which the following liquids can be lifted by means of a siphon: water, oil of density 0·8. The height of the barometer at the time

may be taken as 30 in. of mercury. (Density of mercury = 13·6.) (N.)

(**9**) Describe and explain the action of a simple form of barometer. What effect, if any, on the reading is produced by (*a*) inequalities in the bore of the tube, (*b*) inclining the tube to the vertical? For what reason is mercury chosen as the liquid in such an instrument? (L.)

(**10**) Briefly describe and explain the principle of the action of (*a*) a simple mercury barometer, (*b*) an aneroid barometer. Give a diagram of each.

If the height of a barometer is 75·0 cm. of mercury, what is the pressure of the atmosphere in gm. weight per sq. cm.? Take the density of mercury as 13·6 gm. per c.c. What would be the corresponding height of a water barometer? (N.)

(**11**) Describe a mercury barometer, and explain how it measures the pressure of the atmosphere.

A barometer reads 740 mm. at sea-level. What will it read if it is carried to the top of a mountain 800 m. high? (Take the average density of the air to be 0·00124 gm. per c.c. Density of mercury = 13·6 gm. per c.c.) (O. & C.)

(**12**) The pressure of the atmosphere is said to be 14·7 lb. per sq. in., and also to be 30 in. of mercury. Explain what these statements mean, and point out the relation between them.

A cylindrical vessel 6 in. deep and 12 in. in diameter is evacuated of air. Calculate the crushing force exerted on it by the atmosphere. (O.)

(**13**) Draw graphs to show how the volume of a mass of gas kept at constant temperature varies with the pressure.

The following readings (which represent heights above the table) were taken with a Boyle's law apparatus on a day when the barometric height was 74·8 cm.

Top of closed limb	*Mercury level in closed limb*	*Mercury level in open limb*
35·0 cm.	13·2 cm. 21·8 cm.	16·5 cm. 76·0 cm.

Use these observations to illustrate Boyle's law. (C.)

(**14**) State *Boyle's law.* Give a diagram of a "Boyle's law apparatus" and explain how you would use it to test the truth of this law.

In an experiment to verify Boyle's law two sets of results were :

Volume of air when compressed (in c.c.) .	17·6	12·1
Mercury pressure, in excess of the atmospheric pressure, compressing the air (in cm.) .	10·0	50·0

Calculate the atmospheric pressure at the time of the experiment. (N.)

(**15**) Give a careful sketch of the apparatus you would employ to verify Boyle's law for pressures less than atmospheric, explain how you would use it and show how you would tabulate your readings.

A flask whose volume is 450 c.c. contains air at a pressure of 20 lbs. per sq. in. and is connected to another flask by a piece of india-rubber tubing which is closed by a screw-clip. The second flask has a capacity of 300 c.c. and contains air at a pressure of 15 lbs. per sq. in. What will the pressure in the flasks be after the clip has been opened, if the temperature of the air in the flasks is always the same ? (D.)

(**16**) State Boyle's law for gases, and describe briefly how you would verify it experimentally.

A capillary tube, sealed at one end, contains air enclosed by a thread of mercury 10 cm. long. When the tube is horizontal the column of air is 25 cm. long. What would you expect the length of the air column to be when the tube is held vertically with the open end upwards ? (Height of barometer = 76 cm. of mercury.) (O. & C.)

(**17**) State the relation between the pressure and density of a gas at constant temperature.

A gas cylinder contains 40 litres of gas at 20 atmospheres pressure. Some of the gas is used and the pressure falls to 3 atmospheres. What volume of gas at atmospheric pressure has been used? (W.)

(**18**) An air bubble at the bottom of a lake 272 ft. deep has a volume of 0·1 cub. in. What will be its volume on

reaching the surface, if the barometric pressure is 30 in. of mercury?

(Sp. gr. of mercury is 13·6.) (N.)

(**19**) Describe how you would set up a mercury barometer and use it to determine the pressure of the atmosphere in lbs. per sq. inch or dynes per sq. cm.

A mercury barometer when set up reads 74 cm. but when it is lowered into the mercury until the space above the mercury in the barometer is halved the reading becomes 72 cm. Explain this and calculate the correct atmospheric pressure. (D.)

(**20**) A diving bell of volume 9 cub. m. is lowered into water until the water level inside is about 17 ft. below the external level. If no extra air has been pumped into the bell, what is then the approximate volume of the air inside?

[Approximate height of "water barometer" = 34 ft.] (N.)

(**21**) Describe an experiment to show how the pressure increases with the depth below the free surface of water.

The free surface of water in a diving-bell is 20 metres below the surface of the sea. The atmospheric pressure is 75 cm. of mercury. What will be the height of a barometer in the diving bell? [Sp. gr. of mercury is 13·6, and of sea water is 1·025.] (C.)

(**22**) A diving-bell of capacity 200 cub. ft. rests on the bottom of a dock containing fresh water to a depth of 42·5 ft. If the barometer stands at 30 in. of mercury (sp. gr. 13·6), find how many cubic feet of air at atmospheric pressure must be pumped into the bell to expel all the water from it. (N.)

(**23**) Describe and explain the action of a pressure gauge suitable for the measurement of the pressure of (*a*) the domestic gas supply, (*b*) the steam in a boiler. (L.)

(**24**) How is *pressure* measured in the laboratory? Describe how you would measure the pressure in a vessel partly evacuated by an air-pump.

How would you prove that pressure applied to a liquid is transmitted equally in all directions?

What is the total force on the surface of the mercury in

a barometer reservoir if the area of the surface is 20 sq. cm. and the barometer stands at 760 mm. ?

[Specific gravity of mercury = 13·6.] (O.)

(**25**) How would you measure the pressure of the coal-gas supply in a laboratory ?

If the pressure of the gas supply in the basement of a building is 4 in. of water, what will be the pressure of the same supply at a point 80 ft. higher than the former if the relative densities of gas, air, and water are as 1 : 2 : 1500 ? (W.)

(**26**) Draw a diagram of a Bramah (hydraulic) press and explain, with reference to such a machine, what are meant by *mechanical advantage*, *velocity ratio* and *efficiency*.

In such a press a load of 8 tons is lifted when an effort of 15 lb. is applied on the end of the handle whose leverage is 14 to 1. If the efficiency of the press is 90 per cent., find (*a*) the mechanical advantage, (*b*) the ratio of the area of the ram to the area of the plunger. (O. & C.)

(**27**) Draw a diagram, and explain the action, of a common pump for raising water from a well.

Water is to be lifted from a well 60 ft. deep to a tank 30 ft. above the ground. Describe with a diagram a pump that would be suitable for this purpose. At what height above the water in the well would the pump itself be placed ? (O. & C.)

(**28**) Draw a diagram of a hand-operated pump designed to raise water from a well to the top of a high building, and show how the jerkiness of the flow may be lessened.

In an engineering journal recently the description of a pumping station ran as follows : "The new plant is designed to pump (*a*) 24 million gallons a day against a head of 400 feet, or (*b*) 32 million gallons against a head of 200 feet." At first sight there seems to be a discrepancy in these figures. Can you detect it, and do you consider the statement to be an error ? Give reasons.

Calculate the horse-power represented by the figures given in (*a*) above. (A.)

(**29**) Describe and explain, with a sketch, the inflation of a cycle tyre tube by means of an ordinary cycle pump.

If the tube has a volume, when inflated, of 250 cub. in., and the barrel of the pump has a volume of 10 cub. in., how many strokes of the pump will be necessary in order to inflate the tube to a pressure of 35 lb. per sq. in.? (The pressure of the atmosphere may be taken as 15 lb. per sq. in.) (N.)

(**30**) Describe (with the aid of a diagram) the construction and mode of action of a pump suitable for pumping the air out of a vessel of about 5 litres content.

A certain pump has a barrel whose volume is 200 c.c., and it is being used to exhaust a vessel of volume 2 litres, which contained air initially at a pressure of 60·5 cm. of mercury. Assuming that the operations take place at constant temperature, find the pressure in the vessel after two exhausting strokes of the pump. (O.)

CHAPTER III

The Principle of Archimedes

Archimedes' Principle. c. 250 B.C.

A body wholly or partly immersed in a fluid experiences an upward thrust equal to the *weight* of the fluid which it is displacing.

The *volume* of the fluid displaced is of course equal to the *volume* of that part of the body immersed in it.

Experimental Verification

1. The length and diameter of a metal cylinder are measured. Hence its volume and the weight of displaced liquid can be calculated.

This calculated result is then compared with the difference in readings obtained (i.e., the upward thrust or apparent loss of weight) when the cylinder is weighed suspended in air and then in the liquid.

Thus :—

Wt. of Water displaced.		**Upward Thrust.**	
Length of cylinder	= 5·0 c.m.	Weight of cylinder in air	= 66·45 gm.
Diameter of cylinder	= 1·4 c.m.		
∴ Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times (\cdot 7)^2 \times 5$	= 7·7 c.c.	Weight of cylinder in water	= 58·75 gm.
		∴ Loss of weight or upward thrust	= **7·7** gm.
∴ Volume of water displaced	= 7·7 c.c.		
∴ Weight of water displaced	= **7·7** gm.		

If a liquid, other than water is used, it is necessary to know its density.

2. The apparatus, fig. 23, consists of an upper vessel and a lower cylinder so made that the external volume of the cylinder is equal to the internal volume of the vessel into which the cylinder will fit exactly.

The apparatus is hung from the arm of a balance, and weights are added sufficient to restore the balance.

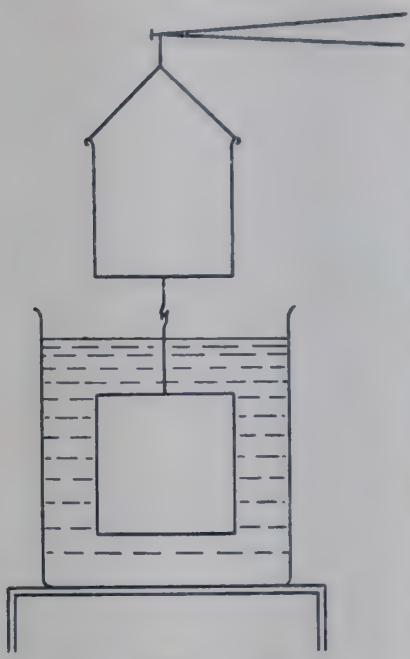

FIG. 23.

The piston is now immersed in a beaker of water which rests on a bridge to keep it clear of the balance pan.

The upward thrust on the piston causes the left hand balance arm to rise. Water is added through a pipette into the cylinder and when it is just full (i.e., when water equal in volume to the piston has been added) the equilibrium is once more just restored.

Centre of buoyancy

This is the centre of gravity of the displaced fluid, before it has been displaced.

For a regular solid of uniform density, its centre of gravity and centre of buoyancy will coincide, when the solid is completely immersed.

A body immersed in a fluid will be acted upon by two forces, its weight vertically downwards through its centre of gravity, and the upward thrust vertically upwards through its centre of buoyancy.

The body will rise or sink depending on which of these forces is the larger and the result will not depend on depth provided that the body is rigid and that the density of the displaced fluid is the same at all depths. (See page 14.)

Further, the body will turn unless the two forces act in the same vertical line, and if totally submerged it will not be in a stable position unless the centre of gravity is *below* the centre of buoyancy. Fig. 24 represents the forces acting on a body weighted at one end.

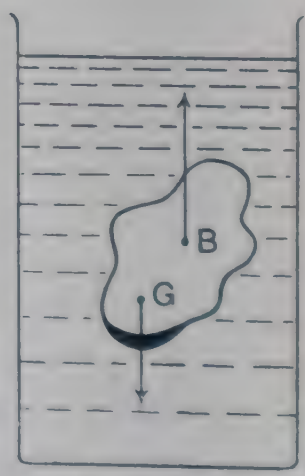

FIG. 24

The Downward Thrust on the Fluid

As an immersed solid is being thrust upwards by the fluid, the solid must react on the fluid by an equal and opposite force downwards.

This can be shown experimentally if a solid, suspended from a spring balance, is weighed in air, and then in water which is contained in a beaker resting on a balance pan.

The spring balance registers the apparent loss of weight of the body in water and the equal apparent increase in the weight of the beaker of water is registered on the scales.

Applications of Archimedes' Principle

1. The specific gravity of a solid

Weigh solid in air . . . . W_a

Weigh solid in water . . . W_w

The apparent loss of weight $W_a - W_w$ is equal to the *weight* of water displaced. This has a *volume* equal to the *volume* of the solid immersed.

$\therefore$ Sp. gr. of solid

$$= \frac{\text{Wt. of solid}}{\text{Wt. of equal volume of water}} = \frac{W_a}{W_a - W_w}$$

2. The specific gravity of a liquid

If the same solid is weighed in air W_a, in water W_w and then in the given liquid W_l, then $W_a - W_l$ is equal to the weight of liquid displaced which also has a volume equal to the volume of the solid.

Hence sp. gr. of liquid

$$= \frac{\text{Wt. of any volume of liquid}}{\text{Wt. of equal volume of water}} = \frac{W_a - W_l}{W_a - W_w}$$

3. The specific gravity of a solid lighter than water

A sinker or piece of heavy substance is used so that when tied to the body it will make it sink in water.

Three weighings are necessary.

Weight of body (such as cork or paraffin wax) in air $= W_1$
Weight of body and sinker together in water $= W_2$
Weight of sinker alone, in water $= W_3$

$\therefore$ Weight of body in air + sinker in water $= W_1 + W_3$.

Hence $(W_1 + W_3) - W_2$ = difference between weight of body in air and in water
= Weight of water displaced by body.

$$\therefore \text{Sp. gr. of body} = \frac{W_1}{W_1 + W_3 - W_2}$$

4. **Balloons and Airships.**

It should be remembered that when a body is weighed in air, by means of a spring balance, the weight recorded is actually the weight of body (in vacuo) *minus* the weight of the fluid (air) displaced.

When a beam balance is used this statement does not apply strictly for the weights themselves displace some air.

In comparison with the weights of ordinary solid bodies the weight of air displaced is usually negligibly small; but balloons and airships are supported by the upthrust of the displaced air.

Buoyancy of an Airship

For an airship 600 ft. long and 70 ft. diameter, if assumed to be cylindrical, the volume will be

$$\frac{22}{7} \times (35)^2 \times 600 \text{ cu. ft.} = 2{,}310{,}000 \text{ cu. ft.}$$

1 cu. ft. of air weighs . . ·08 lb.
1 cu. ft. of hydrogen weighs . ·0056 lb.

$\therefore$ Weight of hydrogen in the airship

$= 2{,}310{,}000 \times \cdot0056$ lb. $= 12{,}936$ lb.

and weight of air displaced

$= 2{,}310{,}000 \times \cdot08$ lb. $= 184{,}800$ lb.

Hence the lifting power

$= 184{,}800$ lb. $- 12{,}936$ lb. $= 76\frac{1}{2}$ tons approx.

If the weight of the fabric, machinery, etc., is, say, 50 tons, the airship could carry a further weight of $26\frac{1}{2}$ tons.

Hydrogen is dangerous because it is inflammable. Helium, a gas which is twice as heavy as hydrogen, is not inflammable, but is much more expensive.

The extra weight, in the above example, due to using helium would be another 12,936 lb. or 5·8 tons, and the lifting power of the airship would be reduced by this relatively small amount.

5. Floating Bodies

Archimedes' Principle applies to bodies which float as well as to those submerged in a fluid. If a body is floating, the upthrust of the liquid on it must equal the weight of liquid it is displacing, since this upthrust just prevents it sinking further into the fluid.

It will sink to such a level that the volume of fluid displaced (which will be equal to the volume of body submerged) has a weight equal to the weight of the body in air.

In other words a body will float in a fluid if its density is less than that of the fluid.

Thus : *Total volume of body* × *Density of body* = Weight of body = Weight of fluid displaced = *Volume of body submerged* × *density of fluid.*

$$\therefore \frac{\text{Immersed part of volume of body}}{\text{Total volume of body}} = \frac{\text{Density of body.}}{\text{Density of fluid.}}$$

If the body is floating in water this ratio is the specific gravity of the body.

Examples.

1. A rectangular block of wood of height 5 in. sinks to a depth of 3 in. below water.

$$\text{Hence its specific gravity} = \frac{3}{5} = 0{\cdot}6.$$

$$\text{In a liquid of sp. gr. } 0{\cdot}8 \qquad \frac{\text{Portion under fluid}}{\text{Total volume}} = \frac{{\cdot}6}{{\cdot}8}$$

Thus it will sink until $\frac{3}{4}$ of 5 in. = $3\frac{3}{4}$ in. is below the surface of the liquid.

2. The specific gravity of ice is 0·912 and of sea water is 1·026.

If v is the volume of an iceberg (of total volume V), which is *under* the surface of the sea, then :

$$v \times 1{\cdot}026 = V \times {\cdot}912$$

$$\therefore \quad \frac{v}{V} = \frac{{\cdot}912}{1{\cdot}026}$$

The fraction of the volume of the iceberg *above* the surface of the sea will therefore be

$$\frac{V - v}{V} = 1 - \frac{v}{V} = \frac{1{\cdot}026 - {\cdot}912}{1{\cdot}026} = \frac{{\cdot}114}{1{\cdot}026} = \frac{1}{9}$$

Hydrometers

An ordinary 6 in. narrow test tube with a flattened end (to give uniform cross section) has some lead shot inside it to make it float vertically.

A strip of stiff lined paper may be placed inside the tube for use as a scale.

Small cylinders of wood soaked in paraffin wax may be used as an alternative if some lead is embedded in one end. They have the advantage that a scale is more easily marked on them, but the disadvantage that they are more difficult to clean when required for different liquids.

In first liquid, fig. 25*a*

$$\text{Wt. of hydrometer} = \text{Wt. of liquid displaced} = l_1 \times a \times d_1$$

where a = cross section of tube or rod

d_1 = density of liquid

In second liquid, fig. 25*b*

$$\text{Wt. of hydrometer} = l_2 \times a \times d_2$$

where d_2 = density of second liquid.

$$\therefore l_2 \times a \times d_2 = l_1 \times a \times d_1$$

$$\therefore \quad \frac{d_2}{d_1} = \frac{l_1}{l_2}$$

Thus if first liquid is water, then sp. gr. of second liquid

$$= \frac{\text{Length of tube in water}}{\text{Length of tube in liquid}}$$

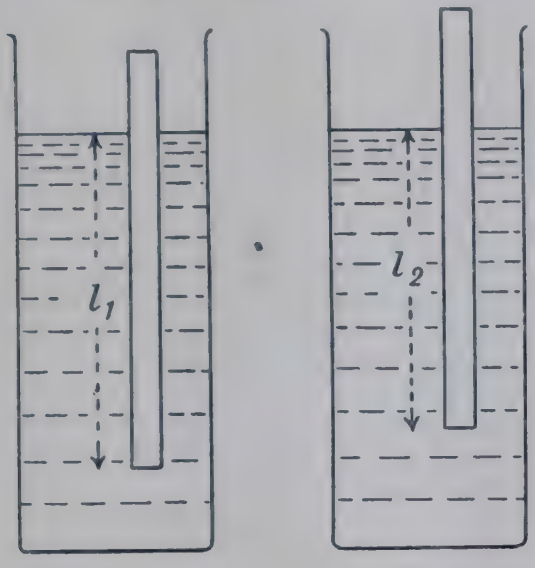

FIG. 25 (*a*) FIG. 25 (*b*)

The Common Hydrometer

A uniform rod is rarely used as a hydrometer because the specific gravity marks are close together ; fig. 26*a*.

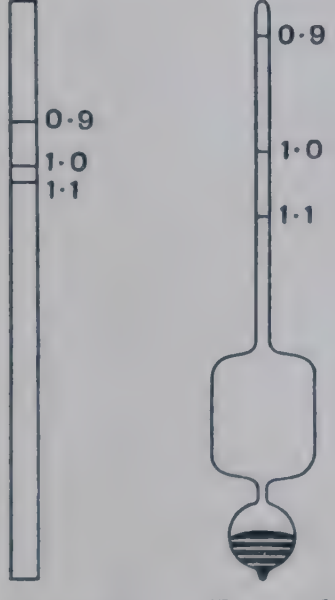

FIG. 26*a* FIG. 26*b*

To avoid this a bulb with a thin stem is used and is weighted to float vertically.

$$\text{Since } \frac{\text{Volume below water}}{\text{Volume below liquid}} = \frac{\text{Density of liquid}}{\text{Density of water}}$$
$$= \text{S.G. of liquid,}$$

the use of a stem of small cross-section will result in the graduations being widely spaced ; fig. 26*b*.

Nicholson's Constant Volume Hydrometer. Fig. 27.

This consists of a hollow metal cylinder with a small pan, (for carrying weights), attached to it by a narrow stem. Below the cylinder is a small cone (weighted with lead). whose top forms another small pan.

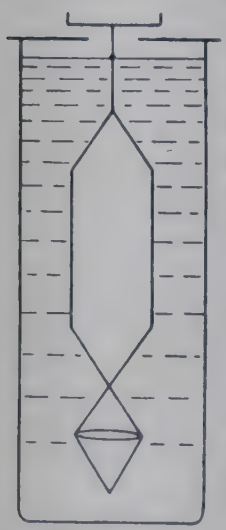

FIG. 27

There is a scratch on the stem to give a fixed point to which the hydrometer can be immersed. A sheet of metal or card with a slit in it is usually placed on the mouth of the container to prevent the hydrometer from sinking if overloaded.

To find the specific gravity of a solid

To sink the hydrometer to the scratch on the stem in water in each case :—

Weights required $= W_1$

Weights required when solid (e.g., glass) is in top pan $= W_2$

$\therefore$ Weight of solid in air $= W_1 - W_2$

Weights required when solid (in water) is in lower pan $= W_3$

$\therefore$ Weight of solid in water $= W_1 - W_3$

$\therefore$ Upward thrust or weight of equal volume of water

$= \text{Weight in air} - \text{Weight in water}$

$= W_1 - W_2 - (W_1 - W_3)$

$= W_3 - W_2$

$$\therefore \text{Sp. gr. of solid} = \frac{W_1 - W_2}{W_3 - W_2}$$

To find the specific gravity of a liquid

For this the weight of the Hydrometer itself is required W
Weight required to sink it to the scratch in water . W_1
Weight required to sink it to the scratch in given liquid . . . . . W_2

The same *volume* is displaced in each case

$$\therefore \text{Sp. gr. of liquid} = \frac{\text{Wt. of this volume of liquid}}{\text{Wt. of this volume of water}} = \frac{W + W_2}{W + W_1}$$

Further Examples on Archimedes' Principle

1. The weight of a solid immersed in water is 3 oz. When immersed in a liquid of specific gravity 1·2 its weight is 2·5 oz. Find what its weight will be in a liquid of specific gravity 1·5.

Let W oz. = weight of solid in air. Let x oz. = Weight of solid in liquid sp. gr. 1·5

Then $W - 3$ = Wt. of equal volume of water
= (say) w oz. . . . . (*a*)

$W - 2{\cdot}5$ = Wt. of equal volume of liquid sp. gr. 1·2
= 1·2 w . . . . . (*b*)

$W - x$ = Wt. of equal volume of liquid sp. gr. 1·5
= 1·5 w . . . . . (*c*)

From equations (*b*) — (*a*) we get $\cdot 5 = \cdot 2\,w \therefore w = 2{\cdot}5$

$$\therefore W = 3 + 2{\cdot}5 = 5{\cdot}5$$

$\therefore$ from equation (*c*) $5{\cdot}5 - x = 1{\cdot}5 \times 2{\cdot}5 = 3{\cdot}75$

$$\therefore x = \mathbf{1{\cdot}75\ oz.}$$

2. A piece of wood weighing 30 gm. floats in water with $\frac{4}{5}$ of its volume immersed.

Find the volume and the density of the wood.

Let V c.c. = Volume of the wood.

Then $\frac{4}{5}$ V c.c. = Volume immersed
= Volume of water displaced.

$\therefore \frac{4}{5}$ V gm. = Weight of water displaced and this balances the weight of the floating wood = 30 gm.

$$\therefore \text{ V } = \frac{5}{4} \times 30 = 37{\cdot}5 \text{ c.c.}$$

$\therefore$ Density of wood $= \frac{30}{37{\cdot}5}$ gm. per c.c. = **0·8 gm. per c.c.**

3. The specific gravities corresponding to the lowest and highest graduations on the stem of a common hydrometer are 1·40 and 1·00 respectively. Find the specific gravity of a liquid in which this hydrometer will float with half its graduated stem immersed.

Let V c.c. = Vol. of hydrometer below the lowest graduation.

Let l cm. = Length of stem between the lowest and highest graduation.

Let a sq. cm. = area of cross section of stem.

In each case the weight of liquid displaced = Weight of hydrometer W.

$\therefore$ W $= (\text{V} + \text{la}) \times 1 = \text{V} \times 1{\cdot}4 = (\text{V} + \frac{1}{2}\text{la}) \times x$

where x = required sp. gr.

$\therefore \cdot 4\,\text{V} = \text{la}$ and $\text{V}(1{\cdot}4 - x) = \frac{1}{2}\text{la}\,x = \frac{1}{2} \times \cdot 4\text{V} \times x$

$\therefore 1{\cdot}4 - x = \cdot 2x \quad \therefore 1{\cdot}2\,x = 1{\cdot}4 \quad \therefore x = \mathbf{1\frac{1}{6}}.$

Questions on Chapter III

(**1**) Describe the experiment you would carry out to verify Archimedes' principle for the case of a heavy solid in water.

A mass of metal weighs 145 gm. in air and 123 gm. in water. What would it weigh immersed in a liquid of specific gravity 0·78? (W.)

(**2**) What is meant by the *specific gravity of a body*? Describe an accurate method of finding the specific gravity of (*a*) a penny, (*b*) petrol.

A metal ball weighs 20·826 grams in air and 18·486 grams in water.

What is its specific gravity?

What weight would it have in a liquid of specific gravity 0·843? (O.)

(**3**) State the principle of Archimedes. Describe an experiment which shows how the principle can be applied in the determination of the specific gravity of (*a*) a solid, (*b*) a liquid.

A piece of aluminium is weighed in air, water, and paraffin oil, the weights being 4·0, 2·5, and 2·8 ounces respectively. Find the specific gravity of aluminium and of paraffin oil. (N.)

(**4**) Distinguish between *density* and *specific gravity*.

If you were provided with a tangle of wire of known diameter, how would you proceed to determine the length of the wire without unravelling it? (L).

(**5**) A bar of aluminium of uniform cross-sectional area 4 sq. cm. is fastened at one end to a spring balance, graduated to read in grams, and hangs vertically. The bar is 10 cm. long and the specific gravity of aluminium is 2·7. Find the reading of the spring balance.

A gas jar of water is brought under the bar and raised until 1 cm. of the length of the bar is immersed in the water. Find the new reading of the spring balance.

Plot a graph showing the relation between the length of the bar immersed and the reading of the spring balance.

Find from your graph what length of the bar is immersed when the reading of the spring balance is 80 gm. (O.)

(**6**) A tank of water rests on a weighing machine which registers 50 lb. wt. A mass of metal is suspended from a spring balance which registers 7 lb. wt. If the specific gravity of the metal is 8·4, what will be the readings of weighing machine and spring balance when the metal is lowered into the water? (W.)

(**7**) A sphere of wood is counterpoised on a common balance by means of brass weights under the receiver of an air-pump when the pressure inside the receiver is atmospheric. As air is pumped out of the receiver the equilibrium is destroyed. Explain why this is so and state clearly which side of the balance will go down. (L.)

(**8**) A piece of tin weighing 20 gm. and a piece of copper are fastened to the two ends of a string passing over a pulley and hang in equilibrium when entirely immersed in water. Determine the weight of the copper.

[S.G. of tin 7·3 ; of copper 8·9.] (O. & C.)

(**9**) State the Principle of Archimedes and deduce from it the Law of Flotation.

A piece of wax of sp. gr. 0·90 weighs 27·00 gm. in air. When 3·38 gm. of copper are fastened to it and it is placed in water, it just floats when both wax and copper are completely immersed. Find the sp. gr. of the copper. (N.)

(**10**) State the Principle of Archimedes and show how it is applied to a floating body. Illustrate this by reference to (*a*) a submarine, (*b*) an airship. (L.)

(**11**) How would you verify by experiment that a floating body displaces its own weight of the liquid in which it is floating? Explain clearly why a tank made of iron plates may float on water although a lump of iron of the same weight sinks in water.

A cubical block of glass, density 2·4 gm. per c.c., encloses an air bubble. The glass weighs 146·4 grams in air and 82·4 grams when immersed in water. What is the volume of the bubble? (D.)

(**12**) What is the lifting power of an airship filled with hydrogen if its volume is 10,000,000 litres, if the density of hydrogen is 0·09 kilo per cub. m. and that of air is 1·2 kilo per cub. m.?

Explain how the lifting power will be affected as the airship rises. (N.)

(**13**) A balloon inflated with hydrogen supports a gross weight of 3,000 lb. If helium were used in place of hydrogen what weight could it then support if the relative densities of hydrogen, helium, and air are as 1 : 2 : 14? (W.)

(**14**) State Archimedes' principle and explain how you would verify it experimentally.

A string will just bear a pull of 200·4 grm. Find the volume of the largest piece of cork (sp. gr. 0·24) which it can hold beneath the surface of mercury (sp. gr. 13·6). (N.)

(**15**) State Archimedes' principle and show how you would apply it to determine the specific gravity of a solid which is not soluble but floats in water.

A rectangular barge 20 ft. long, 6 ft. wide and 3 ft. deep weighs 800 lb. When it is floating unloaded in water, where is the water line? What is the greatest load which can be placed in it without its sinking?

(The weight of 1 cubic foot of water is 62·5 lb.) (D.)

(**16**) Assuming a man whose weight is 60 kilograms to be just able to float in fresh water with his body completely immersed, find how much cork of sp. gr. 0·25 would be required to construct a life-belt which would float him with one-fifth of his volume out of water. (O. & C.)

(**17**) A cylindrical oil drum, 16 inches high and 10 inches in diameter, weighs 6 lb. when empty. It is filled completely with oil of specific gravity 0·92, sealed up and dropped into a pool of water. Will it float? Neglect the thickness of the walls of the drum.

It is not an uncommon belief that a ship which is sunk at sea, where the depth is very great, will be prevented from reaching the bottom because of the great pressure existing there. State whether you consider that this opinion is well-founded, giving reasons for your answer. (A.)

(**18**) A piece of iron sinks in water, yet an iron ship can remain afloat. Explain this.

What volume of lead, of density 11·5, must be attached to a block of wood of volume 500 c.c. and density 0·5, so that the combination will just sink in water? (W.)

(**19**) State Archimedes' principle and show how it is applied to find the specific gravity of (*a*) a piece of iron, (*b*) a piece of wood.

A piece of waxed wood of uniform section is weighted so as to make it float vertically in a liquid. In milk it floats with 40 cm. of its length immersed but in oil of sp. gr. 0·9 it has 48 cm. immersed. What is the specific gravity of the milk? (N.)

(**20**) Define *specific gravity.*

The closed end of a test tube is flattened and the tube weighted with lead shot so that it floats upright in water.

If 10 cms. of the tube are immersed when the tube floats in distilled water, how far will it sink in a liquid of specific gravity 0·8?

If the mass of the weighted tube is $31\frac{2}{5}$ grams, what is the diameter of the test tube? ($\pi = 3{\cdot}14$.)

Explain each step in your calculation. (L.)

(**21**) Describe the common hydrometer and explain the method of graduating it.

The stem of a hydrometer is graduated upwards from 0 to 100 in equal parts. The volume of the graduated stem is one-quarter of the volume of the instrument below the 0 mark. When placed in water it sinks to the 20 mark. Find the density of the liquid in which it sinks to the 0 mark. (O. & C.).

(**22**) A wooden rod of square section measures 2 cm. along each side, and when placed in a certain oil floats so that 5 cm. of the rod are above the surface and 19 cm. are below. When floating in water 16 cm. of the rod are immersed. Find the density of (*a*) the oil, (*b*) the wood. What is the weight of the rod?

Why is the stem of a hydrometer always made so that it has a small cross-section? (D.)

(**23**) State the principle of Archimedes and describe **one** method of verifying it experimentally.

A hydrometer has a total volume of 70 c.c., and its upper end is a uniform cylinder 11 cm. long and of cross-sectional area 0·8 sq. cm. When placed in water it floats with a length of 10 cm. out of the water, and when placed in a certain liquid it floats with a length of 1·4 cm. out of the liquid. Calculate the weight of the hydrometer and the specific gravity of the liquid. (O.)

(**24**) Under what conditions does a body float partially immersed in a liquid?

A common hydrometer has a volume of 40 c.c. up to the 1·0 mark, and a stem with an area of cross-section of 0·6 sq. cm. If the stem is graduated for a distance of 20 cm. below the 1·0 mark, what range of specific gravities will the instrument measure? (O. & C.)

(**25**) A common hydrometer of mass 35 gm. floats in

water with 4 cm. of its stem above the surface, the area of cross-section of the stem being 0·5 sq. cm. What length of stem will be above the surface when the hydrometer is floating in a liquid of specific gravity 1·25? (C.)

(**26**) A cylindrical glass beaker, of internal diameter 5 cm., height 7 cm. and mass 38·5 gm., floats upright in water. Paraffin (sp. gr. 0·8) is carefully poured into the beaker until it is on the point of being submerged. Neglecting the thickness of the glass, find the height of the paraffin in the beaker.

[Take π as 22/7.] (N.)

(**27**) Describe a form of hydrometer suitable for measuring both the density of a liquid and the density of a solid body. In each case explain how the measurements are taken.

A solid body is placed on the upper pan of a Nicholson's hydrometer floating in water and shot is added until the instrument sinks to the mark. The body is then removed, and 5·85 grams are found to be required to sink the instrument to the mark. When the body is placed in the lower pan, 3·683 grams are necessary to sink the instrument to the mark. Calculate the density of the body, explaining each step you take. (O.)

(**28**) On what factors does the pressure at the bottom of a vessel containing a liquid depend? Explain the difference between the thrust on, and the pressure at the bottom of a vessel containing a liquid.

A hollow cylinder, which is closed at one end, is fitted with a light airtight piston whose area is 40 square inches. The piston encloses beneath it 460 cubic inches of air at atmospheric pressure and the whole apparatus weighs 12 lb. Calculate the depth to which it must be immersed in water so that, when released, it will sink. Neglect the thickness of the cylinder walls and the friction of the piston.

Take the height of the barometer as 30 inches and the specific gravity of mercury as 13·6. (A.)

HYDROSTATICS

Rapid Revision Test

The density of a substance is defined as (**1**) whereas its specific gravity is defined as (**2**). Thus the density of water on the English system is (**3**) while on the metric system it is (**4**). The specific gravity of water is (**5**). The weight of any substance can be calculated from its specific gravity, its volume and the density of water. Thus a stone of volume 1·2 c.ft. and S.G. 2·5 will weigh (**6**). Conversely volumes can be found. A ball of external diameter 21 cm., weighing 300 gm. and made of rubber of S.G. 1·5 will contain (**7**) c.c. of rubber so that the volume of air in it is (**8**).

To measure the density of a piece of glass the simplest method is (**9**). Specific gravity bottles are generally used for liquids and for powders. The formula which gives the S.G. of a liquid is (**10**). For the determination of the S.G. of a powder some observations were : Weight of powder 105 gm. Weight of bottle full of water 52·7 gm. Weight of bottle full of powder and water 120·2 gm. Hence the S.G. of the powder is (**11**).

The pressure at a point in a liquid is defined as (**12**). Its value is given by the product (**13**). The thrust on a plane surface immersed in a liquid is calculated from the formula (**14**). The pressure exerted by the water at a tap enables us to find the height of the surface of the water in the reservoir measured from the level of the tap. If the pressure is 25 lb. per sq. in. above atmospheric pressure this height is (**15**).

A rapid method of finding roughly the S.G. of a liquid is by Hare's apparatus illustrated in (**16**). The calculation is (**17**).

A hydraulic press makes use of the fact that a pressure applied to any point in a fluid is transmitted to all points in the fluid. In a press of which one piston is 1 inch in diameter and the other 10 ins., a force of 40 lb. applied to the smaller piston will set up on the other a thrust of (**18**).

When the height of the mercury barometer is 76 cm. the pressure of the atmosphere is (**19**) in gm. per sq. cm. and (**20**) in lb. per sq. inch.

Boyle first stated the law which connects the volume of a gas with its pressure. This law is (**21**). The general equation for the behaviour of a quantity of gas is (**22**). The air in a glass tube one yard long closed at the top will be compressed when the tube is lowered vertically into water. When the top of the tube is level with the surface of the water, the water inside will have risen through (**23**), on a day when the height of the water barometer is 34 feet. A faulty barometer reads 75 cm. when it should read 76. The length at the top of the tube into which some air has leaked is 10 cm. The atmospheric pressure on a day when the faulty instrument reads 74 cm. will be (**24**).

Diagram (**25**) shows a water pump suitable for drawing water from a well 100 ft. deep. (**26**) shows the pump of a fire engine. An air pump to compress air is shown in (**27**) and one used to take air out of a vessel in (**28**).

Archimedes discovered (**29**). His principle can be illustrated by an experiment shown diagrammatically in (**30**). The principle is useful if it is necessary to measure accurately the volume of a small body of irregular shape, such as a gold ring. The procedure is (**31**). The expt. gives also the S.G. of the ring as (**32**). The S.G. of any liquid is found by Archimedes' principle by (**33**), and the S.G. of a solid soluble in water by (**34**).

A lump of iron weighing 1540 gm. and of S.G. 7·7 is hanging from a spring balance. It is lowered into a bucket of water supported on another spring balance. The reading of the latter is 1000 gm. before the weight enters the water. When the weight is completely submerged the readings of the two balances are (**35**) and (**36**).

An airship floats when its own weight, including the gas in it, equals (**37**). A boat floats so long as (**38**). If a body floats with $\frac{1}{4}$ of its volume above water its S.G. is (**39**). An iceberg of S.G. 0·9 in sea water of S.G. 1·02 has (**40**) of its volume below the surface.

A hydrometer suitable for measuring the S.G. of the acid in an accumulator is shown in diagram (**41**). If its weight

is 24 gm. it displaces (**42**) c.c. of water. It will displace (**43**) c.c. of a liquid of S.G. 1·2. If the change of level is 8 cm. the area of cross section of the stem of the hydrometer is (**44**).

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HEAT

CHAPTER IV

Heat and Temperature

Heat is one of the forms of **energy**, that is, it is capable of doing **work**. This is illustrated by its effects.

Effects of Heat

(*a*) It usually produces *expansion* in size of varying amount when applied to solids, liquids and gases.

(*b*) It may produce a *change of state*, e.g., ice → water → steam.

(*c*) It may produce a rise of *temperature* (see below).

Other Forms of Energy

(1) Potential Energy—the energy possessed by a body owing to its position,

e.g., a coiled spring, a car at the top of a hill.

(2) Kinetic Energy—the energy possessed by a body owing to its motion,

e.g., a bullet, a stream of water, a car in motion.

(3) Electrical Energy,

e.g., in electric batteries, from dynamos.

(4) Chemical Energy,

e.g., in gunpowder, petrol, and similar substances.

(5) Light Energy,

e.g. from the Sun.

Temperature is the measure of the hotness or coldness of a body. It can be thought of as the condition of a body which determines whether heat energy will pass to it or from it when it is placed in contact with another body.

Thus *temperature* corresponds in many respects to *difference of pressure* in hydrostatics, and to *difference of potential* in electricity.

Scales of Temperature

Temperatures are measured by *thermometers*, depending usually for their action on the effect which heat has on the size of substances.

FIG. 1

Thus a rise of temperature of *one degree Centigrade* is such that it causes the mercury of a mercury-in-glass thermometer to expand through $\frac{1}{100}$th of the distance which it expands between the melting point of pure ice and the boiling point of pure water.

Melting Points and Boiling Points are recorded at standard atmospheric pressure.

These points may be marked or checked on a thermometer as follows :—

The Lower Fixed Point

The bulb is immersed in pure melting ice, contained in a funnel, with the level of the liquid in the stem just visible above the ice. (Fig. 1.)

After 20 minutes or so the level is steady and a mark is made.

The Upper Fixed Point (for a mercury thermometer)

The bulb and stem of the thermometer are surrounded by the steam from pure boiling water.

The steam passes through a second glass jacket before escaping. (Fig. 2.)

If the atmospheric pressure, at the time of the experiment, is not standard, viz. 760 mm. of mercury, then a correction for the effect of pressure on the boiling point must be made.

N.B.—This method cannot be used for an alcohol thermometer (see page 54).

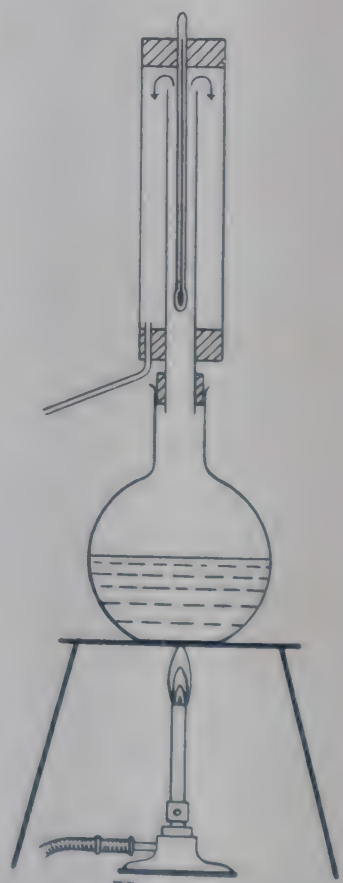

FIG. 2

Scales of Temperature	*Melting point of pure ice*	*Boiling point of pure water*
Centigrade Scale .	0° C.	100° C.
Fahrenheit Scale .	32° F.	212° F.
Réaumur Scale .	0° R.	80° R.

Conversion of temperatures from one scale to another

If C, F, and R are corresponding temperatures on the above scales it follows that :

$$\frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{R - 0}{80 - 0}$$

or

$$\frac{C}{100} = \frac{F - 32}{180} = \frac{R}{80}$$

or $C = \frac{5}{9}(F - 32)$, $F = \frac{9}{5}C + 32$, $C = \frac{5}{4}R$, etc.

Also

difference between two temperatures on the Fahrenheit Scale $\Big)$ $=$ $\Big\{$ $\frac{9}{5}$ths of the difference between two corresponding temperatures on the Centigrade Scale.

Example

At what temperature is the reading of a Fahrenheit thermometer equal to double that of a Centigrade thermometer ?

For such a temperature $F = 2C$

$$\therefore \frac{C}{100} = \frac{2C - 32}{180}$$

$$\therefore 9C = 10C - 160$$

$$\therefore C = \mathbf{160°}$$

and $F = \mathbf{320°}$

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Mercury as a thermometric liquid

(*a*) It is easily obtained pure and is easy to read.
(*b*) It is a good conductor of heat.
(*c*) It has a convenient range. F.P. −40°C., B.P. 360° C.
(*d*) It does not " wet " the glass.
(*e*) Very careful experiments have shown that mercury expands very regularly.

Alcohol is sometimes used in thermometers

(*a*) Range of temperatures. F.P. −130° C., B.P. 78° C.
(*b*) For a given rise of temperature it expands much more than mercury does.

But :

(*c*) It is a poor conductor.
(*d*) It does " wet " the glass.
(*e*) Careful experiments have shown that its expansion is not quite uniform.

As alcohol boils at 78° C., the upper fixed point of an alcohol thermometer cannot be fixed at 100° C. Its graduations should be made by comparison with a mercury thermometer.

Gas Thermometers. (— 150° C. to 1400° C.), containing air, nitrogen, or hydrogen, make use of the known relation between volume and temperature of a gas at " constant pressure," or between pressure and temperature at " constant volume." (See page 77.)

Electrical Thermometers. (— 250° C. to 3000° C.), depend either on the variation of the electrical resistance of a wire with temperature, or on the voltage produced when the temperatures of the junctions between a pair of wires differ.

Six's Maximum and Minimum Thermometer

This is essentially an alcohol thermometer. The expansion of the alcohol forces the long thread of mercury round the tube.

Small light dumb-bell shaped indexes are pushed to the maximum and minimum positions by the convex mercury

surfaces in the right and left limbs of the U-tube as shown in fig. 3.

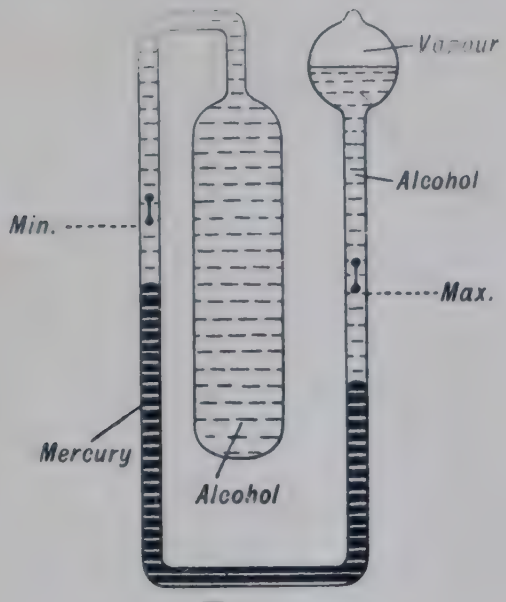


FIG. 3

Each index usually has an iron wire inside it so that it can be moved by a magnet.

Clinical Thermometer

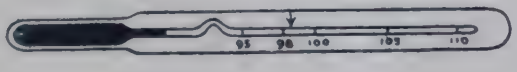


FIG. 4

This is a " maximum " mercury-in-glass thermometer reading from 95° F. to 110° F.

A narrow constriction just above the bulb prevents the mercury in the stem from returning to the bulb, when the latter cools.

The mercury has to be shaken down past the constriction before the instrument is ready for use. Fig. 4.

Questions on Chapter IV

(1) Define the temperatures 0° C. and 100° C.

The stem of a mercury thermometer is of uniform bore and has a centimetre scale marked on it. The lower

fixed point reading is 2·2 cm. and the upper fixed point reading 20·4 cm. What is the temperature (*a*) in ° C., (*b*) in ° F., when the top of the mercury thread indicates 5·1 cm. ? (L.)

(**2**) Define the *upper fixed point* of a temperature scale.

Describe, giving diagrams, how you would find where to mark both the upper and lower fixed points on an ungraduated mercury thermometer if you did not possess another thermometer.

Why is glass not an ideal material for the bulb of a thermometer ?

(3) Explain the Centigrade and Fahrenheit scales of temperature.

" A hot-water heating system produces a rise in temperature of 41° F. in water entering the system at 63° F."

Re-write the above sentence using the Centigrade in place of the Fahrenheit scale of temperature. (W.)

(**4**) At what temperature are the readings of a Centigrade and Fahrenheit thermometer the same ?

(**5**) Describe briefly the construction and graduation of an *alcohol* thermometer to read temperatures from 20° F. to 120° F.

Express these temperatures on the Centigrade scale. (L.)

(**6**) Describe and explain the action of a thermometer suitable for measuring the minimum night temperature in a greenhouse.

Clinical thermometers are generally graduated to indicate a range of temperature from 95° F.—110° F. What would be the corresponding temperatures in degrees Centigrade ? (O. & C.)

(**7**) Explain why water is not a suitable liquid for use in thermometers. How are the upper and lower fixed points of a thermometer determined ? What reading on a Centigrade thermometer corresponds to 77° on the Fahrenheit scale ? (O.)

CHAPTER V

Expansion of Solids

Solids increase in size when their temperatures are raised. The amount of expansion depends on the original size, the rise of temperature, and the material of which the solid is made. Thus metals in general expand more than non-metals, and of the metals, copper expands about half as much again as iron for equal lengths and rises in temperature.

We are usually concerned with expansions in length.

The coefficient of Linear Expansion of a substance is the increase in length *per unit length* for 1° Centigrade rise in temperature.

N.B. If a Fahrenheit scale of temperature is used the coefficient per 1° F. will obviously be only $\frac{5}{9}$ths of the above.

Thus :

L_t = Length of rod of substance at $t°$ C

L_o = Length of rod of substance at $o°$ C.

Then coefficient of Linear Expansion $= \dfrac{L_t - L_o}{L_o \times t} = \alpha$

$$\therefore L_t - L_o = L_o \times t \times \alpha$$

$$\therefore \quad L_t = L_o (1 + \alpha t).$$

Measurement of Coefficient of Linear Expansion

The increase in length is very small and for accurate work this is measured by means of a micrometer screw gauge, or magnified by a system of levers.

The student should be able to describe *in detail* the particular method which he has used.

Example

A rod of copper 50 cm. long is found to increase in length

by ·072 cm. when its temperature is raised from 15° C. to 100° C.

∴ 50 cm. of copper rising 85° C. increase by ·072 cm.

∴ 1 cm. of copper rising 1° C. increase by $\frac{\cdot 072}{50 \times 85} = \cdot 000017$.

The strictly accurate definition of the coefficient should read *per unit length at* 0° *C.*

It will be noted that in the above example we have divided by 50 the length at 15° C.; strictly we should divide by the length at 0° C. which would be about 49·99, but this would not appreciably alter the result.

Even if we divided by the length at 100° C., viz. 50·072, the answer, to two significant figures, would not be affected.

Applications of Knowledge of Coefficients of Expansion

1. Measurements made by metal chains in surveying have to be corrected for expansion due to changes of temperature.

2. The metal rim of a wooden cart-wheel if put on hot will " shrink-on " and produce a tight fit when cold.

Similarly boiler plates, if rivetted together with red hot rivets, will be pulled tightly together as the rivets cool.

3. The glass stopper in a bottle may often be loosened by pouring hot water on to the outside of the neck of the bottle. The neck thus expands before the stopper. Glass is a poor " conductor " of heat and some time elapses before the stopper itself becomes heated.

For similar reasons when hot water is poured into a *thick* tumbler the inside of the glass expands producing such a state of strain that the glass often cracks.

4. " Gaps " are left between adjacent steel railway lines to prevent buckling due to expansion. The size of the gap will depend on the maximum temperature variation expected.

Similarly, roller bearings or some other device must be used, to take up expansion, in the construction of long metal bridges.

5. *The Compensated Pendulum*

If the " length " of a pendulum increases then it swings slower and loses time.

(a) *Graham's Mercury Pendulum.* (Fig. 5.)

The iron rod carries a hollow bob containing mercury.

The amount of mercury is adjusted so that when the temperature rises the rise in the centre of gravity of the mercury due to expansion just counteracts the effect of the expansion of the iron.

FIG. 5

(b) *Harrison's Gridiron Pendulum.* (Fig. 6.)

This consists of 5 steel rods and 4 brass rods.

It is symmetrical and as far as expansion is concerned the " effective lengths " are 3 steel rods expanding downwards and 2 brass rods expanding upwards.

FIG. 6

The coefficient of expansion of steel is ·000012 and of brass ·000020. The length of each rod necessary to keep the effective length of the pendulum constant can thus be calculated.

Invar

An alloy containing 36% nickel and 64% steel has practically no coefficient of expansion and can be used to make a non-expanding pendulum.

N.B. Another nickel-steel alloy containing 45% nickel has the same coefficient of expansion as glass and can therefore be used for carrying an electric current into a glass vacuum bulb.

If a hot metal wire is pushed through softened glass, either a loose fit or cracked glass will be obtained unless the two coefficients of expansion are the same. The very expensive metal platinum has also the same coefficient of expansion as glass. It was formerly used when wires were sealed into glass.

(c) *The Compensated Balance Wheel of a Watch.* (Fig. 7.)

The rim of the wheel is composed of strips of two metals, the outer one having the larger coefficient of expansion.

It forms an incomplete circle of 2 or 3 sectors attached to separate radii.

On expansion the radii expand but this is counteracted by the sectors becoming more curved. The small weights, adjustable, on the free ends of the sectors thus move rather nearer to the centre of the wheel.

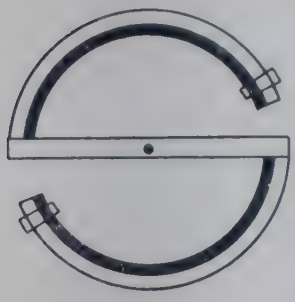


FIG. 7

A rise of temperature decreases the elasticity of the spring and the watch loses time for this reason also.

Hence the balance wheel must be "overcompensated" for its own expansion, and to make it of invar would not suffice.

6. When taking accurate measurements with a barometer, allowance must be made both for the effect of temperature on the density of mercury, and for the expansion of the scale.

Coefficients of Surface and of Volume Expansion

Let A_o = Area of a surface at 0° C.

and A_t = Area of this surface at t° C.

Then coefficient of surface (or superficial) expansion is defined thus :— $\beta = \dfrac{A_t - A_o}{A_o \times t}$.

Hence $A_t = A_o (1 + \beta t)$.

Let V_o = Volume of a substance at 0° C.

and V_t = Volume of a substance at t° C.

Then coefficient of volume (or cubical) expansion is defined thus :— $\gamma = \dfrac{V_t - V_o}{V_o \times t}$

Hence $V_t = V_o (1 + \gamma t)$.

Connection between α, β, and γ.

Let l_o = length of a side of a cube at 0° C.

and l_t = length of a side of this cube at t° C.

Then $l_t = l_o (1 + \alpha t)$.

$\therefore \quad l_t^3 = l_o^3 (1 + \alpha t)^3 = l_o^3 (1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3)$
$= l_o^3 (1 + 3\alpha t)$ neglecting $\alpha^2 t^2$ and $\alpha^3 t^3$ which are very small quantities since α is small

But $l_o^3 = V_o$ and $l_t^3 = V_t$

Hence $V_t = V_o (1 + 3\alpha t)$.

Therefore, to a close approximation $\gamma = 3\alpha$.

Similarly it can be shown that $\beta = 2\alpha$ (approximately).

For glass $\alpha = \cdot 000009$ and $\gamma = \cdot 000027$

Worked Examples

1. A distance is read as 76·20 cm. when measured by a brass scale which is correct at 0° C. What is the true length if the reading was taken at 15° C. ?

[Coefficient of linear expansion of brass = 0·000019.]

Each 1 cm. on scale at 0° C. becomes

$1 + (15 \times \cdot 000019)$ cm. at 15° C.

$= 1 \cdot 000285$ cm. at 15° C.

Hence at 15° C. the reading 76·20 cm. is read in terms of scale divisions which are larger than true cms. Hence this reading is equivalent to $76 \cdot 20 \times 1 \cdot 000285$ true cms.

$= \mathbf{76 \cdot 22}$ **cm.** Ans.

2. An iron pipe, of coefficient of expansion ·000012, is 500 cm. long at 100° C. What will be its length at 10° C. ?

1 cm. of iron at 10° C. will become at 100° C.

$1 + (90 \times \cdot 000012)$ cm.

$= 1 \cdot 00108$ cm.

$\therefore$ 500 cm. of iron at 100° C. are equivalent to

$\frac{500}{1 \cdot 00108}$ cm. at 10° C.

$= \mathbf{499 \cdot 46}$ **cm.** Ans.

Questions on Chapter V

(**1**) What is meant by the coefficient of linear expansion of a solid body ? Describe in detail any **one** method of finding it for a metal such as iron.

An iron bridge is 80 ft. long. How much clearance would it be necessary to leave at one end to allow for the expansion of the bridge between the limits of temperature — 5° C. to 25° C.? (N.)

(Coefficient of linear expansion of iron = 0·000012.)

(**2**) Define the *coefficient of linear expansion* of a solid. In carrying out a measurement of this coefficient for brass the number 0·000018 was obtained when centimetres and degrees C. were used. What would have been the result if inches and degrees F. had been employed?

Describe two practical applications of the contraction of metals in cooling. (L.)

(**3**) A steel tire has a diameter of 99·7 cm. at 15° C. To what temperature must it be raised to enable it to be put on to a wheel 100 cm. in diameter? (O. & C.)

(Coefficient of linear expansion of steel = 0·000012 per °C.)

(**4**) Give **two** examples of devices used to overcome difficulties produced by the expansion of substances by heat. Explain the principles on which they are based.

Iron rods, *AB* and *AC*, 3 and 4 cm. long respectively, are joined at right-angles at *A*. What must be the coefficient of expansion of the material forming the side *BC* of the triangle if it is to remain right-angled at higher temperatures? (O.)

(Coefficient of linear expansion of iron = 0·000011.)

(**5**) Describe how you would measure accurately the increase in length of a brass bar or tube when heated from room temperature to 100° C., giving full details of the experiment and a sketch of the apparatus.

A circular steel band, whose internal diameter when measured at 20° C. is 59·8 cm., has to be slipped over a metal cylinder whose diameter is 60·36 cm. To what temperature must the band be raised in order that this may be done? Take the coefficient of linear expansion of steel to be 0·000012 for 1° C. (A.)

(**6**) Show how the coefficient of superficial expansion of a solid is related to its coefficient of linear expansion. An iron plate having an area of 5 sq. cm. just fits into a hole in a brass plate at 15° C.

What is the area of the gap between the two plates at 100° C.? Coefficient of linear expansion of iron 0·000012, of brass 0·000018. (B.)

(7) Prove that the coefficient of volume expansion is approximately three times the coefficient of linear expansion.

Describe, with a diagram, the compensated balance wheel of a watch, and explain how it enables the watch to keep correct time at varying temperatures.

A rod is measured at 25° C. by a brass scale which is correct at 0° C., and is found to be 51·00 cm. long. What is the true length of the rod at 25° C.? (O.)

[Coefficient of linear expansion of brass = 0·000018.]

CHAPTER VI

Expansion of Liquids

The expansion observed when a liquid is heated depends partly on the expansion of the liquid and partly on that of the vessel which contains it.

The coefficient of "apparent" or "relative" expansion of a liquid is the apparent increase in volume for 1° C. rise in temperature divided by the volume at 0° C.

The coefficient of "real" or "absolute" expansion of a liquid is the real increase in volume for 1° C. rise in temperature divided by the volume at 0° C.

It can be shown that :

Coefficient of real expansion = *Coefficient of apparent expansion* plus *coefficient of cubical expansion of container* (*i.e.*, 3 × *coefficient of linear expansion of container*).

Measurement of Coefficient of Apparent Expansion

(a) *Bulb and Stem Method*

(i) The apparatus is calibrated thus :—

It is weighed empty, then with the bulb filled with water, and finally with the bulb and a length of stem (measured in centimetres) also filled with water.

By subtraction, and assuming that 1 gm. of water occupies 1 c.c., we now know the volume of the bulb (measured to any fixed mark), and the volume represented by each 1 cm. length of the stem (assumed to be uniform).

(ii) The bulb and stem containing a convenient amount of the given liquid is now immersed in melting ice at 0° C. and the level of the liquid is marked.

It is then raised to some known temperature and the expansion recorded.

Using the calibration values (i) above, the coefficient of apparent expansion of the liquid can be calculated.

(b) *Specific Gravity Bottle Method*

A clean dry specific gravity bottle with stopper is weighed $= W$ gm.

It is then filled with the given liquid (e.g., glycerine or turpentine) at *room* temperature t° C. $= W_t$ gm.

Hence weight of liquid $= W_t - W$ gm.

If d = density of the liquid at this room temperature in gm. per c.c., then its volume $= \frac{W_t - W}{d}$ c.c.

The bottle and its contents is then immersed in a beaker of boiling water at 100° C. until the liquid ceases to expand. It is then removed, wiped dry and weighed again after it has cooled to room temperature $= W_{100}$.

Weight of contained liquid now $= W_{100} - W$ gm.

Its volume at *room* temperature $= \frac{W_{100} - W}{d}$ c.c.

At 100° C. this volume of liquid just fills the bottle of volume $\frac{W_t - W}{d}$ c.c.

(for apparent expansion the volume of bottle is assumed to be the same at both temperatures).

Hence coefficient of *apparent* expansion

$$= \frac{\text{Increase in Volume}}{\text{Volume at t° C.} \times \text{Rise in temp.}}$$

$$= \frac{\frac{W_t - W}{d} - \frac{W_{100} - W}{d}}{\frac{W_{100} - W}{d}(100 - t)}$$

$$= \frac{W_t - W_{100}}{(W_{100} - W)(100 - t)}.$$

or, in words, $= \frac{\text{Weight expelled}}{\text{Weight } \textit{left in} \times \text{Rise in temperature}}$

N.B. For the strict definition, the temperature t° C. should be 0° C.

(c) *Weight Thermometer Method*

The measurements and calculation required are precisely the same as when the specific gravity bottle is used.

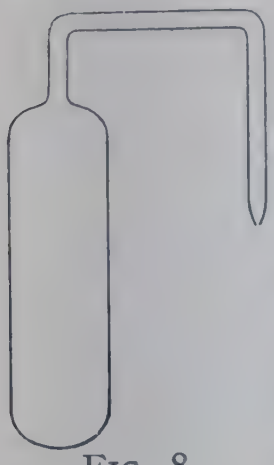


FIG. 8

The weight thermometer, fig. 8, is rather more difficult to fill with liquid.

It can readily be made from an ordinary test tube.

The relation between the density of a liquid at different temperatures and its coefficient of cubical expansion

Let V_o and D_o be the volume and density respectively at 0° C., and let V_t and D_t be the volume and density respectively at t° C.

The *weight* of liquid is unaffected by temperature and

$$= V_o \times D_o = V_t \times D_t \quad \therefore V_t = \frac{V_o \times D_o}{D_t}$$

Coefficient of expansion

$$= \frac{V_t - V_o}{V_o \times t} = \frac{\dfrac{V_o \times D_o}{D_t} - V_o}{V_o \times t}$$

$$= \frac{D_o - D_t}{D_t \times t} = \frac{\text{Change in density}}{\text{Density at } \textit{higher} \text{ temp.} \times \text{Rise in temp.}}$$

Measurement of real or absolute coefficient of expansion.

One of the properties of the pressure at a point in a column of liquid is that it is proportional to the *depth* of the point and independent of the cross-sectional area of the column. (See Hydrostatics, page 12.)

The densities of two liquids can therefore be compared by measuring the relative heights of balancing columns, that is, columns in which the pressures at the top and bottom are the same in each liquid. (See Hare's Apparatus.)

Columns of the *same* liquid at two different temperatures may be similarly balanced.

The U-tube method, fig. 9, is often used.

Let H_o = height of the cold column

and H_t = height of the hot column.

Then the hot column produces at the bottom of the U tube a pressure $H_t \times D_t$ which is balanced by the equal pressure $H_o \times D_o$ due to the cold column

$$\text{i.e. } H_t \times D_t = H_o \times D_o \text{ or } D_o = \frac{H_t \times D_t}{H_o}$$

Hence the coefficient of real or absolute expansion

$$= \frac{V_t - V_o}{V_o \times t} = \frac{D_o - D_t}{D_t \times t} \text{ (see above)} = \frac{\dfrac{H_t \times D_t}{H_o} - D_t}{D_t \times t}$$

$$= \frac{H_t - H_o}{H_o \times t}$$

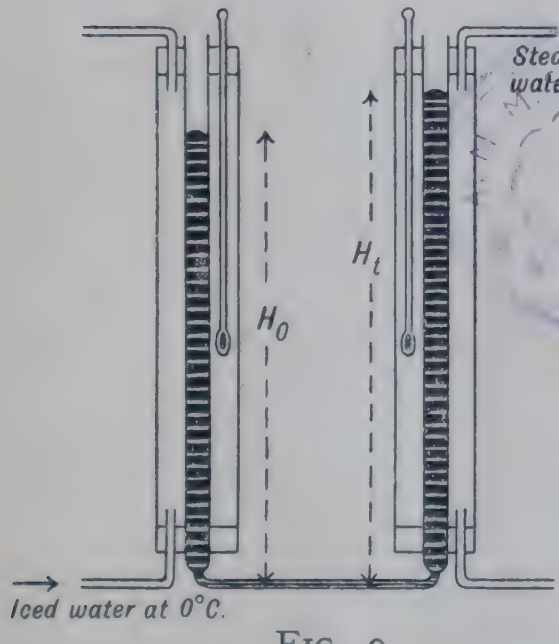


FIG. 9 .

Note that since we are balancing two pressures and since these pressures depend only on the heights of the liquid

columns the result is independent of any changes in the cross sections of the tubes, and so the expansion of the U tube does not affect the result.

The difference $H_t - H_0$ is small and must therefore be measured accurately (to $\frac{1}{1000}$ cm.) by using a travelling telescope or "cathetometer."

The irregular expansion of water

The volume of a given mass of water at 0° C. *contracts* as its temperature is raised to 4° C., at which temperature it has a minimum volume or *maximum density*.

Above 4° C. the volume increases as the temperature is raised.

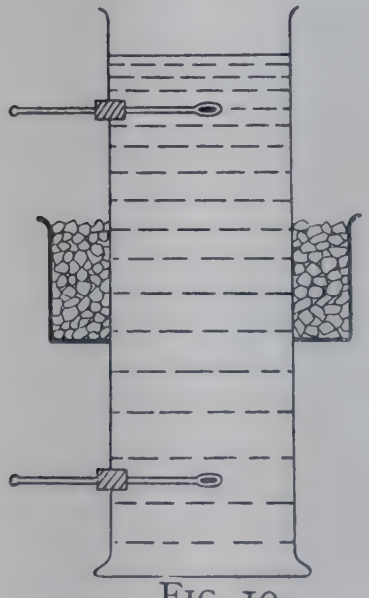


FIG. 10.

This fact is illustrated by **Hope's Apparatus**, fig. 10, which consists of a cylinder containing water originally at room temperature. A trough round the middle of the cylinder is filled with freezing mixture and the temperatures of the top and bottom of the column of water are recorded by thermometers inserted through the wall of the cylinder.

Water near the middle becomes colder and therefore denser than elsewhere, and so sinks to the bottom. This continues until the temperature at the bottom reaches 4° C. When all the water below the middle has been cooled to 4° C., any further cooling causes the cold water to rise and so the temperature at the top, which has so far remained fairly steady, begins to fall rapidly. This fall continues until 0° C. is reached, when ice forms on the surface, although the lower thermometer is still recording 4° C.

These results are shown in fig. 11, where the temperatures are plotted against the time.

Hence ice forms on the surface of lakes and rivers and if they are reasonably deep the water near the bottom never falls below 4° C.

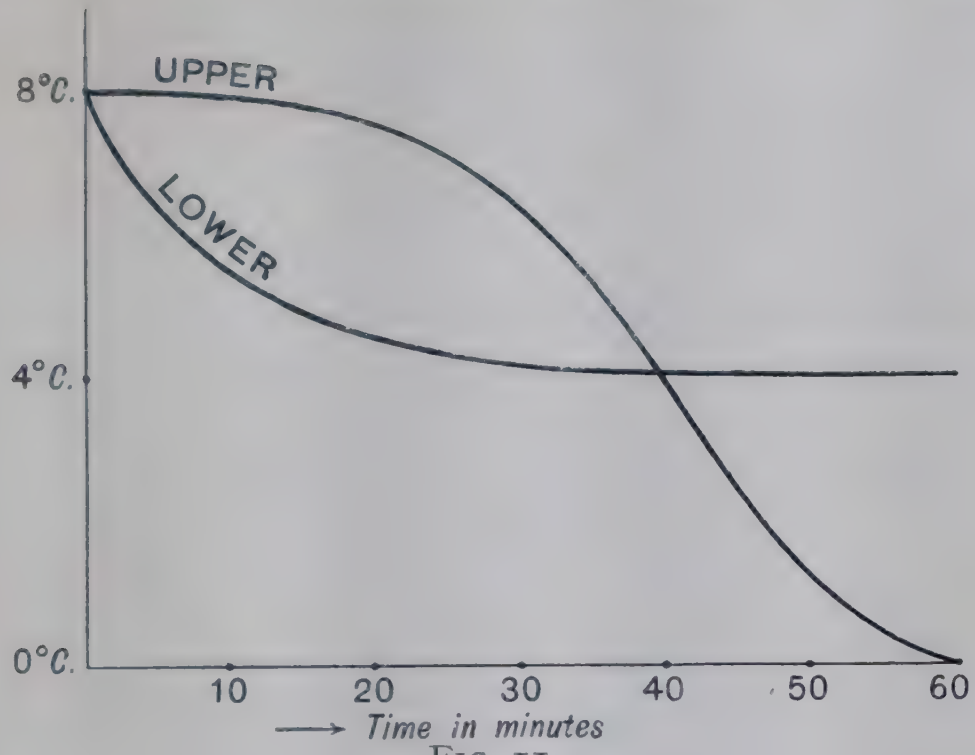


FIG. 11.

Worked Examples

1. The volume of a bulb is 100 c.c. and 10 cm. of the attached stem represent a volume of 2 c.c.

A certain liquid occupies the bulb and 2·4 cm. of the stem at 0° C. and when the temperature is raised to 50° C. it expands so as to occupy the bulb and 15·5 cm. of stem.

Calculate the coefficient of apparent expansion of the liquid.

Each 1 cm. of stem is equivalent to $\frac{2}{10}$ c.c. = 0·2 c.c.

$\therefore$ Volume of liquid at 0° C. = $100 + (2{\cdot}4 \times {\cdot}2) = 104{\cdot}8$ c.c.

Expansion $= (15{\cdot}5 - 2{\cdot}4) \times {\cdot}2 = 2{\cdot}62$ c.c.

$\therefore$ Coefficient of apparent expansion

$$= \frac{2{\cdot}62}{104{\cdot}8 \times 50} = \mathbf{{\cdot}00050.}$$

2. A specific gravity bottle weighs 20·54 gm. when empty, 65·27 gm. when full of turpentine at 15° C. and 62·54 gm. when full of turpentine at 80° C.

Calculate (*a*) the coefficient of apparent expansion, (*b*) the absolute coefficient of expansion of the turpentine,

assuming that the coefficient of linear expansion of the glass of which the bottle is made is 0·000009.

Weight of turpentine at 15° C.

$$= 65{\cdot}27 - 20{\cdot}54 = 44{\cdot}73 \text{ gm.}$$

Weight of turpentine at 80° C.

$$= 62{\cdot}54 - 20{\cdot}54 = 42{\cdot}00 \text{ gm.}$$

Weight expelled for 65° C. rise $= 2{\cdot}73$ gm.

$$\therefore \text{Coefficient of apparent expansion} = \frac{2{\cdot}73}{42 \times 65} = \mathbf{{\cdot}001000}$$

Coefficient of cubical expansion of glass

$$= 3 \times {\cdot}000009 = \mathbf{{\cdot}000027}$$

$\therefore$ Absolute coefficient of expansion of turpentine

$$= \mathbf{{\cdot}001027}$$

Questions on Chapter VI

(**1**) Distinguish between real and apparent expansion as applied to liquids, and show what is the relation between them.

A specific gravity bottle weighs 15·44 gm. when empty, and 59·64 gm. when full of a certain liquid at 15° C. It is heated in a bath at 60° C., after which it is found that exactly 1 gm. of liquid has been expelled. Calculate the coefficient of apparent expansion of the liquid.

If the coefficient of linear expansion of the glass is ·000009 per ° C., find the coefficient of real expansion of the liquid. (B.)

(**2**) A specific gravity bottle with a perforated stopper weighs 24·4 grams when empty and 86·4 grams when filled with a certain liquid at 20° C. The bottle and its contents are heated in boiling water until no more liquid escapes through the stopper. When cold the weight of the bottle and its contents are now found to be 84·28 grams. What is the apparent coefficient of expansion of the liquid? (D.)

(**3**) Explain clearly why mercury is used for filling ordinary thermometers.

The volume of the bulb of a mercury-in-glass thermometer up to the 0° C. mark is 2 c.c. The distance between

the fixed points of the thermometer is 30 cm. What is the diameter of the stem of the thermometer?

(Coefficient of expansion of mercury in glass = 0·00015 per ° C.) (O. & C.)

(**4**) The area of the bore of a capillary tube is 0·002 sq. cm. and a thermometer is to be constructed from this tube, the distance between the two fixed points being 25 cm. If the coefficient of expansion of mercury relative to glass is 0·00015 per ° C. and the lower fixed point is at the bottom of the stem, calculate the volume of the bulb required. (L.)

(5) Describe a method for the accurate determination of the coefficient of real expansion of a liquid, and explain the theory of the method used.

A litre bottle full of milk which has just been pasteurised at 60° C. is sealed and cooled to 15° C. Find the volume of the empty space in the bottle at the new temperature, assuming that the coefficients of cubical expansion of milk and glass are 0·00038 and 0·000025 per degree C. respectively. (O. & C.)

(**6**) If the two limbs of a U-tube containing a liquid are kept at different temperatures the levels are not the same in the two limbs. Explain why this is so and show how, by appropriate measurements, the coefficient of expansion of the liquid may be calculated. Is this the coefficient of real or apparent expansion? (C.)

(**7**) Define *coefficient of linear expansion* and *coefficient of cubical expansion*, and prove that the latter is approximately three times the former.

Describe an experiment to show that water has a maximum density at 4° C., and explain **two** natural consequences of this. (O. & C.)

(**8**) Describe an experiment which shows that (*a*) in general liquids expand when they are raised in temperature, and (*b*) equal volumes of different liquids expand by different amounts when they are raised through the same temperature interval.

Describe, with a graph, how water behaves when raised from 0° C. to 16° C., and deduce from your description

the temperature distribution with depth, in a pond during a period of hard frost in winter. (D.)

(**9**) The coefficient of expansion of aniline is 85×10^{-5}. At 15° C. its density is 1·023 gm./c.c.

Calculate the density at 85° C.

The density of water varies in the following manner :—

20°	30°	40°	50°	60°	70°	80° C.
0·998	0·996	0·992	0·988	0·983	0·978	0·972

Plot graphs between the same pair of axes to show how the densities of these two liquids vary, and so find the temperature at which they are equal. (A.)

CHAPTER VII

Expansion of Gases

Experiments show that gases differ from solids and liquids in that :—

(*a*) The gas must be contained in some vessel which is closed and yet allows the gas to expand or contract freely.

(*b*) The expansions are much larger than with solids and liquids.

Hence the relatively small expansion of the containing vessel may be neglected. Further, the denominator in the expression for the coefficient of expansion must certainly be " volume at 0° C."

(*c*) The volume of a given mass of gas varies considerably with pressure changes even when the temperature is constant. (See Boyle's Law.)

Hence : **The coefficient of volume expansion of a gas, at constant pressure, is the increase in volume for 1° C. rise in temperature, divided by the volume at 0° C.**

(*d*) This coefficient is very nearly *the same for all gases*, at temperatures well above their liquefying points ; and is equal to $\frac{1}{273}$ or ·00366.

Measurement of Coefficient of Expansion of Air at Constant Pressure

A piece of uniform glass tubing sealed off at one end is attached to a glass scale. A column of air is enclosed between a small pellet of dry mercury above and another at the closed end (to fill up any irregularity of the sealed end). Fig. 12.

The apparatus is immersed in iced water at 0° C., the length of the column is measured and then the temperature is gradually raised. At each of a number of temperatures, after considerable stirring, the length of air column, between the two pellets, is measured.

FIG. 12

The lengths of air column are proportional to volumes and from these the coefficient is calculated.

If it is not convenient to take a reading at 0° C., the readings may be plotted against the temperatures and the resulting graph, a straight line, if produced back will give the 0° C. reading.

This experiment verifies **Charles' Law** that :— **The volume of a given mass of gas increases by $\frac{1}{273}$ of its volume at 0° C. for each 1° C. rise in temperature provided that the pressure is kept constant.**

Absolute Zero—Absolute Temperatures

V_0 = volume of a given mass of gas at 0° C.

V_t = Volume of a given mass of gas at t° C.

By Charles' Law : —

$$\frac{V_t - V_0}{V_0 \times t} = \frac{1}{273} \qquad \text{or } V_t = V_0\left(1 + \frac{t}{273}\right)$$

$$\text{or } V_t = V_0\left(\frac{273 + t}{273}\right)$$

If this Law were to hold good for all gases at low temperatures, it follows from the above expression that when t° C. reaches — 273° C. then $V_t = 0$.

It is impossible to imagine a gas reduced to a volume less than 0, hence — 273° C. is called the **Absolute Zero** of temperature. This is the lowest possible temperature ; at this temperature no more heat is left in the body.

Temperatures are sometimes measured from this point thus :—

Centigrade	*Absolute*
— 273° C.	0°
0° C.	273°
100° C.	373°
t° C.	T° = 273 + t° C.

Charles' Law may be written then in a simplified form

$$\frac{V_t}{273 + t} = \frac{V_o}{273} \qquad \textit{or} \qquad \frac{V_t}{T} = \frac{V_o}{T_o}$$

i.e., $\frac{V}{T}$ is constant where V is the volume of gas at absolute temperature T, or

The volume of a given mass of gas is directly proportional to its absolute temperature when the pressure is kept constant.

Relations Between P, V, and T

Boyle's Law $P \times V$ is constant if T is constant

Charles' Law $\frac{V}{T}$ is constant if P is constant

It follows that $\frac{P \times V}{T}$ is always constant for a given mass of gas and that if V is constant then $\frac{P}{T}$ should be constant.

The Gas Equation

The equation $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ is often called the Gas Equation since it tells us how the pressure, volume and temperature of a particular mass of gas are related.

Remember that, in this form of the equation, T_1 and T_2 are *absolute temperatures*. Many beginners find it better to remember the equation in the form

$$\frac{P_1V_1}{273 + t_1} = \frac{P_2V_2}{273 + t_2}$$

where t_1 and t_2 are the Centigrade temperatures.

To verify experimentally that, if the volume of a given mass of gas is kept constant, its pressure is proportional to its absolute temperature; or that its pressure increases by $\frac{1}{273}$ of its pressure at 0° C. for every 1°C. rise in temperature.

A glass bulb is joined by *capillary* tube (why?) bent twice at right angles, to a length of flexible rubber tubing. This is connected to a reservoir of mercury which may be

raised or lowered on a wooden scale so as to keep the mercury level at X, and hence the volume of air, constant ; fig. 13.

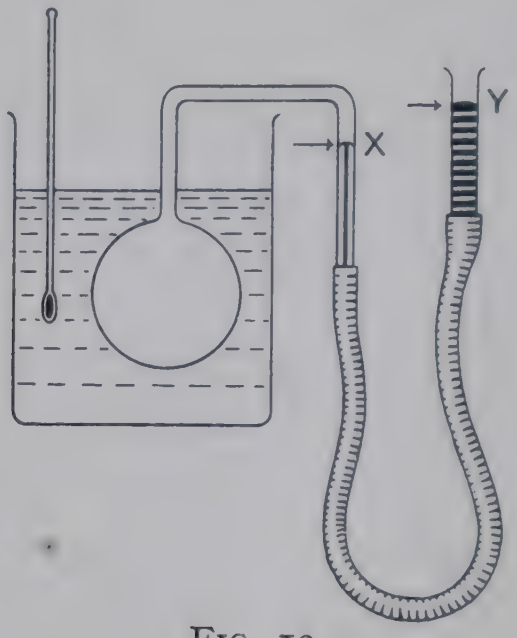


FIG. 13

The pressure at any moment will be the height of Y above X, **plus the atmospheric pressure** also expressed in centimetres of mercury.

The bulb of air is surrounded by a beaker of water whose temperature is read by a thermometer after thorough stirring. A series of corresponding pressures and temperatures are observed.

It is convenient to tabulate them thus :

Atmospheric Pressure = B	*Temp.* t° C.	*Temp. Abs.* T = t + 273	*Differ. of mercury levels* d	*Pressure* = P = B ± d	$\frac{P}{T}$
	.....	.....	.....	.....	.....
	.....	.....	.....	.....	.....
	.....	.....	.....	.....	.....

$\frac{P}{T}$ should give a constant.

Constant volume air thermometer

The apparatus just described, fig. 13, may be used as a thermometer.

If the pressure corresponding to one fixed temperature, such as 0° C. or 100° C., is measured, then the value of $\frac{P}{T}$ so obtained will give the temperature corresponding to any other pressure, since $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Remember that P_1 *and* P_2 *are the total pressures, i.e., barometric pressure plus the pressure difference* YX *and that* T_1 *and* T_2 *are absolute temperatures (i.e.,* 273 + *centigrade temperatures).*

For example see 4 below.

Worked Examples

1. The volume occupied by 32 gm. of oxygen at 0° C. and 760 mm. is 22·4 litres. What volume would it occupy at 15° C. and 740 mm. pressure?

$$\frac{P_1 \times V_1}{T_1} = \frac{P_2 \times V_2}{T_2}$$

$$\therefore \frac{760 \times 22{\cdot}4}{273 + 0} = \frac{740 \times V_2}{273 + 15} \therefore V_2 = 22{\cdot}4 \times \frac{760}{740} \times \frac{288}{273}$$

$$= \mathbf{24{\cdot}27} \text{ litres.}$$

2. A bulb containing air is placed successively in ice, boiling water at 100° C., and a liquid at unknown temperature; the volume is kept constant and the pressure in the three cases is found to be 91 cm., 124·3 cm., and 110 cm. Find, without assuming further data the temperature of the liquid. (B.)

Increase of pressure from 0° C. to 100° C. = 124·3 − 91
= 33·3 cm.

Increase of pressure from 0° C. to t° C. = 110 − 91
= 19·0 cm.

Assuming uniform increase in pressure, we have

$$\frac{t}{100} = \frac{19}{33{\cdot}3} \therefore t = \frac{1900}{33{\cdot}3}$$

$$= \mathbf{57°\ C.}$$

3. The density of air at 0° C. is ·00130 gm. per c.c.

Calculate the density of air, at the same pressure, at 20° C.

Vol. of 1 c.c. at 0° C. becomes

$$1 + \frac{20}{273} \text{ c.c.} = \frac{293}{273} \text{ c.c. at } 20° \text{ C.}$$

Hence weight of 1 c.c. becomes

$$\frac{\cdot 00130}{293/273} \text{ gm. per c.c. at } 20° \text{ C.}$$

$$= \frac{\cdot 00130 \times 273}{293} = \mathbf{\cdot 00121 \text{ gm. per c.c.}}$$

4. A constant volume gas thermometer shows a pressure of 84 cm. of mercury when placed in steam over water boiling at standard atmospheric pressure. What is the temperature of a liquid in which the pressure of this thermometer is 88 cm. of mercury?

Since $P \propto T$ then $84 = k\,(273 + 100)$

and $88 = kT$

$$\text{Hence } T = \frac{88}{84} \times 373 = 390{\cdot}7° \text{ absolute}$$

$\therefore$ Temperature on the Centigrade Scale $= 390{\cdot}7 - 273$

$= \mathbf{117{\cdot}7°}$ C.

Questions on Chapter VII

(1) State the relation connecting the pressure, volume, and temperature of a gas like air.

A quantity of air occupies 250 c.c. at 15° C. when the pressure is 76 cm. of mercury. At what temperature will its volume be 230 c.c. if the pressure is 85 cm.? (W.)

(2) What is the effect of temperature upon a gas (*a*) at constant volume, (*b*) at constant pressure?

Hot air at 700° C. and 30 atmospheres pressure is admitted to an engine. It is expanded to five times its volume and cooled to 200° C. At what pressure will it now leave the engine? (O.)

(3) How does the volume of a given mass of gas vary

when its pressure and temperature are simultaneously changed?

A man requires to breathe the same mass of air per minute at the top of a mountain as at the foot. The respective pressures and temperatures are 72 cm. and 17° C. at the foot and 48 cm. and 2° C. at the top. If he breathes 15 times per minute at the foot, how often must he breathe per minute at the top, assuming that he always fills his lungs to the same extent? (O. & C.)

(**4**) Describe, with the aid of a diagram a constant volume air thermometer.

A thermometer of this kind is being used on a day when the barometer stands at 75 cm. of mercury. The bulb is placed in melting ice and the level of the mercury in the open limb is 5 cm. *lower* than that in the closed limb. Find—

(*a*) the pressure of the gas in the bulb,

(*b*) the temperature of the gas in the bulb when the level of the mercury in the open limb is 65 cm. *above* that in the closed limb. (O.)

(**5**) A bulb *A* containing air is connected by rubber tubing containing mercury to a long vertical closed tube *B* from which the air is exhausted. Initially, when the temperature of *A* is 12° C., the level of mercury in *B* is 30 cm. above that in *A*. When the temperature of *A* is increased, it is found that in order to keep the volume of air constant, the tube *B* must be raised until the difference in the two mercury levels is 35 cm. Draw a diagram of the apparatus and find the second temperature of the air in *A*, explaining clearly the principles involved. (N.)

(**6**) The density of dry air at 0° C. and 760 mm. pressure is 1·29 gm. per litre. What will be the weight of a litre of air at 15° C. and 750 mm. pressure?

(**7**) State what is meant by "coefficient of cubical expansion" and explain why this is not a definite quantity in the case of a gas.

A flask is filled with 4 gm. of a gas at 12° C. and heated to 50° C. Owing to the escape of some of the gas, the pressure in the flask is the same at the beginning and end of the experiment.

Find what weight of gas has escaped. (O.)

CHAPTER VIII

Calorimetry

Unit Quantity of Heat, or 1 Calorie, is defined as the quantity of heat required to raise the temperature of 1 gm. of water 1° C.

Other Units

(*a*) The British Thermal Unit (B.Th.U.) is the quantity of heat required to raise the temperature of 1 lb. of water through 1° F.

(*b*) The Therm is 100,000 British Thermal Units.

(*c*) The Centigrade Heat Unit is the quantity of heat required to raise 1 lb. of water 1° C.

N.B. 1 lb. = 453·6 gm. and 1 Fahrenheit degree = $\frac{5}{9}$th Centigrade degree.

$$\therefore 1 \text{ B.Th.U.} = 453{\cdot}6 \times \frac{5}{9} \text{ cals.} = 252 \text{ cals.}$$

In making calculations on quantities of heat it is usually assumed that no *heat* is lost or gained when substances are mixed.

Example

42 gm. of water at 72° C. are added to 32 gm. of water at 16° C. What will be the final temperature of the mixture?

Let θ° C. be the final temperature.

Then 1 gm. of water falling 1° C. loses 1 cal.

$\therefore$ 42 gm. of water falling $(72 - \theta)$° C. lose $42\,(72 - \theta)$ cals.

Similarly 32 gm. of water rising $(\theta - 16)$° C. gain $32\,(\theta - 16)$ cals.

Assuming no heat to be gained or lost through any other cause then

$$42\,(72 - \theta) = 32\,(\theta - 16)$$

$$\therefore 32\theta + 42\theta = (42 \times 72) + (32 \times 16)$$

$$\therefore 74\theta = 3024 + 512 = 3536$$

$$\therefore \theta = \frac{3536}{74} = \mathbf{47{\cdot}8}° \text{ C.}$$

The Calorimeter and its Water Equivalent

Experiments on quantities of heat, such as the above, are performed in a "Calorimeter" (fig. 14.)

This consists of an inner metal pot, containing a thermometer and stirrer, inside an outer pot. The two are separated by a non-conducting padding of cotton wool, or felt.

The inner pot will affect the result for it will require some heat to change its temperature.

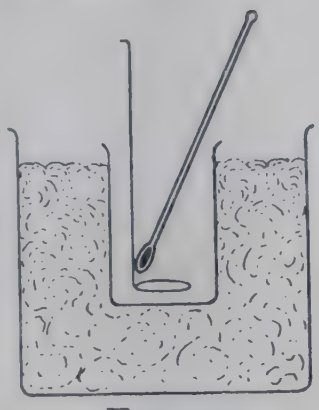


FIG. 14.

The Heat Capacity of a body is defined as the quantity of heat required to raise its temperature 1° C.

Suppose that the Heat Capacity of a calorimeter is x calories per 1° C. Then these x calories could raise x gm. of water 1° C. and thus the calorimeter is in effect equivalent to x *gm.* of water.

The Water Equivalent of a calorimeter is defined as the weight of water which could be raised 1° C. by the heat required to raise the calorimeter 1° C.

Measurement of Water Equivalent

The inner pot of the calorimeter with stirrer is weighed
empty = 50 gm.

It is again weighed when about $\frac{1}{4}$ full of water = 100 gm.
The temperature is observed to be 15·0° C.

Hot water at 46·5° C. is rapidly poured into the calorimeter, the mixture is well stirred and the final temperature is found to be 30·0° C.

The whole is now re-weighed = 150 gm.

Hence hot water added = 50 gm.

Let x = water equivalent of calorimeter.

Heat Lost

50 gm. of hot water falling (46·5 — 30) = 16·5° C. lose $50 \times 16{\cdot}5$ cals.

Heat Gained

50 gm. of cold water rising (30 — 15) = 15° C. gain 50×15 cals.

x gm. (water equivalent) rising (30 — 15) = 15° C. gain $x \times 15$ cals.

Heat gained = Heat lost

$$\therefore (x \times 15) + (50 \times 15) = 50 \times 16{\cdot}5$$

$$\therefore x = \frac{75}{15} = \mathbf{5\ gm.}$$

Specific Heat

Experiments show that equal weights of different materials require different quantities of heat to raise their temperatures 1° C.

Specific Heat of a substance is defined as the *number* of calories required to raise the temperature of 1 gm. of the substance 1° C.

Thus 1 gm. of substance rising 1° C. requires S cals. (S = specific heat).

$\therefore$ M gm. of substance rising θ° C. requires $M \times \theta \times S$ cals. whereas M gm. of *water* rising θ° C. requires $M \times \theta$ cals.

The specific Heat is often defined as the ratio

$$\frac{\text{Heat required to raise unit mass of substance 1°}}{\text{Heat required to raise unit mass of water 1°}}$$

Thus it is a ratio or number. Hence in the definition given above it is stated as the *number* of calories and not as the quantity of heat. This number will be the same whatever pair of consistent heat and temperature units is used.

E.g. The specific heat of iron is 0·12, i.e., 0·12 calories will raise 1 gm. of iron 1° C., or 0·12 B.Th.U.s will raise 1 lb. of iron 1° F.

Relation between specific heat and capacity for heat and water equivalent

In the case of the calorimeter above its water equivalent is 5 gm., and its capacity for heat is 5 cals. Its weight is 50 gm.

Hence to raise the temperature of 50 gm. of it 1° C. requires 5 cals.

∴ to raise the temperature of 1 gm. of it 1° C. requires

$$\frac{5}{50} = 0{\cdot}1 \text{ cal.}$$

Thus Capacity for Heat is equal to

Weight of Substance × Specific Heat

Calorimeters are often made of copper whose specific heat is 0·095 or approx. 0·1.

∴ The water equivalent of a calorimeter unless otherwise stated is often taken as $\frac{1}{10}$th of its weight.

Measurement of Specific Heat

A lump of metal is weighed. Weight = W gm.

It is suspended together with a thermometer in some form of dry heater, such as a corked boiling tube immersed in a can of water which can be boiled. It remains there until its temperature is as near 100° C. as possible.

In the meantime a calorimeter is prepared, its inner pot and stirrer is weighed alone, Weight = m gm. and again after about 50 c.c. of water have been added. Weight = M gm.

∴ Weight of water = (M − m) gm.

Its temperature is read accurately to $\frac{1}{10}$° C. = t° C.

The hot metal at observed temperature, say T° C, is transferred *as rapidly as possibly* to the water in the calorimeter. This is well stirred and the maximum temperature recorded. Final temperature = say, θ°C.

Let x = specific heat of metal.

Heat Lost

W gm. of metal falling (T − θ)° C. lose W × (T − θ) × x cals.

Heat gained

(M − m) gm. of cold water rising (θ − t)° C. gain

(M − m) × (θ − t) cals.

m gm. of calorimeter (sp. ht. = s) rising $(\theta - t)^\circ$ C. gain

$$m\ (\theta - t) \times s \text{ cals.}$$

$$\text{Heat lost} = \text{Heat gained}$$

$$\therefore W \times (T - \theta) \times x = (\theta - t)\ (M - m + ms)$$

x can thus be calculated.

Sources of Error

(1) Heat is lost whilst the hot metal is transferred, i.e., its temperature when it reaches the calorimeter is lower than T above.

T as used in above expression is therefore too high, hence $(T - \theta)$ is too high and s, as calculated, is *too low*.

(2) During the time taken to observe the final temperature, some heat is lost by radiation, etc. (see Chap. X) from the calorimeter.

Hence θ is lower than it ought to be and therefore $(T - \theta)$ is again too high and also $(\theta - t)$ is too low. Both these will give s, as calculated, *too low*.

The Specific Heat of a Liquid

In the above expression for calculating the specific heat of a metal, s can be obtained provided that the other quantities are known.

If some liquid other than water is used then its specific heat can be measured if a metal of a known specific heat s is used as the hot body.

N.B. Another method for finding the specific heat of a liquid—the method of cooling—is given in Chapter X.

The Temperature of a Bunsen Flame

A piece of metal, such as a penny, may be held in a bunsen flame until it acquires its temperature. It is then dropped into water in a calorimeter. If all the necessary weights and temperatures as above, are noted, then the unknown, T° C., may be calculated (losses of heat on mixing are neglected).

Latent Heat

Heat energy is required to change the state of a substance from solid to liquid or from liquid to gas, even though the

temperature remains constant during the melting or the boiling. Conversely this heat is given out during the reverse change.

For example :—

The Latent Heat of Fusion of Ice is the quantity of heat required to convert 1 gm. of ice at 0° C. into 1 gm. of water at 0° C. This is approximately 80 cals. per gm.

The Latent Heat of Vaporisation of Water (usually called Latent Heat of Steam) is the quantity of heat required to convert 1 gm. of water at 100° C. into 1 gm. of steam at 100° C. This is approximately 536 cals. per gm.

Measurement of Latent Heat of Fusion of Ice

This experiment cannot be performed rapidly. The initial temperature of the calorimeter should therefore be about as much above room temperature as the final temperature will be below it. Heat lost by radiation during early part of experiment will then approximately balance that gained during the later part.

Small pieces of ice are dried by flannel or blotting paper and added one by one to water, which is kept stirred, in a calorimeter. The weight of ice added is found later.

Care should be taken not to cool the water to so low a temperature that dew will form on the outside of the calorimeter. This may produce serious errors in the experimental result.

The necessary observations are shown in the following calculation. (Room temperature about 15° C.)

Weight of calorimeter and stirrer (both copper, sp. ht. = ·1) = 52·4 gm.

Weight of calorimeter and stirrer + tepid water at 22·0° C. = 111·8 gm.

Final temperature after adding dry ice at 0° C. = 8·0° C.

Final Weight of calorimeter and contents = 122·2 gm.

∴ Wt. of ice added = 10·4 gm.

Let L = Latent Heat of Ice.

Heat Lost

59·4 gm. of water falling (22 — 8) = 14° C. lose
$59{\cdot}4 \times 14$ cals.

52·4 gm. calorimeter falling (22 — 8) = 14° C. lose
$52{\cdot}4 \times 14 \times {\cdot}1$ cals.

Total Heat Lost $= 14 \times 64{\cdot}64$ cals.

Heat gained

10·4 gm. of ice at 0° C. becoming 10·4 gm. of water at 0° C. gain $10{\cdot}4 \times L$ cals.

This 10·4 gm. of water at 0° C. rise to 8° C. and gain
$10{\cdot}4 \times 8$ cals.

$$\therefore \text{Total Heat gained} = 10{\cdot}4\,(L + 8)$$

$$\therefore 10{\cdot}4 \times (L + 8) = 14 \times 64{\cdot}64$$

$$\therefore L = \frac{14 \times 64{\cdot}64}{10{\cdot}4} - 8 = 87{\cdot}0 - 8 = \mathbf{79{\cdot}0} \text{ cals. per gm.}$$

Measurement of Latent Heat of Steam

Some modification in the above must obviously be made.

Steam from a tin or flask of boiling water is passed through a steam trap, and then through a narrow short glass tube. The end of this is wiped with filter paper just before the steam is allowed to condense under the surface of water in a calorimeter previously prepared.

When the temperature has risen approximately 20° C. or 25° C. the glass jet is rapidly removed, and the maximum final temperature is then observed accurately, after stirring.

A final weighing gives the weight of steam added.

Example

Weight of calorimeter and stirrer (sp. ht. = ·1)
= 48·40 gm.

Weight of calorimeter and stirrer
+ water at 15·0° C. = 299·60 gm.

Final Temperature of mixture = 38·5° C.

Final weight of calorimeter and contents = 309·74 gm.

∴ Weight of steam added = 10·14 gm.

Let L = Latent Heat of Steam

Heat Lost

10·14 gm. of steam at 100° C. condensing to 10·14 gm. of water at 100° C. lose 10·14 × L cals.

This 10·14 gm. of water fall (100 — 38·5)° C. and lose 10·14 × 61·5 cals.

$$\text{Total heat lost} = 10{\cdot}14\,(L + 61{\cdot}5)$$

Heat Gained

251·2 gm. water rising (38·5 — 15·0)° C. gain 251·2 × 23·5 cals.

48·4 gm. calorimeter and stirrer rising (38·5 — 15·0)° C. gain 48·4 × 23·5 × ·1 cals.

$$\text{Total heat lost} = 256{\cdot}04 \times 23{\cdot}5 \text{ cals.}$$

$$\therefore 10{\cdot}14\,(L + 61{\cdot}5) = 256{\cdot}04 \times 23{\cdot}5$$

$$\therefore L = \frac{256{\cdot}04 \times 23{\cdot}5}{10{\cdot}14} - 61{\cdot}5 = 593\,4 - 61{\cdot}5$$

$$= \mathbf{532} \text{ cals. per gm.}$$

Worked Examples (in addition to those in the text).

1. A mass of copper weighing 50 gm. is placed in a furnace and then rapidly transferred to 250 gm. of water at 10° C. contained in a vessel whose water equivalent is 15 gm. The temperature of the water rises to 21·3° C. Find the temperature of the furnace. (Sp. ht. of copper = ·096.) (C.)

Let θ° C. = required temperature.

Heat lost : $50 \times (\theta - 21{\cdot}3) \times {\cdot}096$ cal.

Heat gained : $(250 + 15)\,(21{\cdot}3 - 10)$ cal.

$$\therefore 50 \times (\theta - 21{\cdot}3) \times {\cdot}096 = 265 \times 11{\cdot}3$$

$$\therefore \theta - 21{\cdot}3 = \frac{265 \times 11{\cdot}3}{50 \times {\cdot}096} = 623{\cdot}9$$

$$\therefore \theta = \mathbf{645{\cdot}2}° \text{ C.}$$

2. A piece of ice at — 20° C. is dropped into a calorimeter containing 100 gm. of water at 35° C. The mass of ice is 30 gm., the water equivalent of the calorimeter is 5 gm. and the resulting temperature is 7·2° C.

Calculate the specific heat of ice. (Latent heat of fusion of ice = 80 cal. per gm.) (O. & C.)

Let s = required specific heat.

Heat lost : $(100 + 5)(35 - 7{\cdot}2) = 105 \times 27{\cdot}8$ cal.

Heat gained : 30 gm. ice rising from $-20°$ C. to $0°$ C. gain $30 \times 20 \times s$ cal.

30 gm. ice at $0°$ C. becoming water at $0°$ C. gain 30×80 cal.
This 30 gm. water at $0°$ C. rising to $7{\cdot}2°$ C. gain $30 \times 7{\cdot}2$

Total heat gained $= 30(20s + 87{\cdot}2)$ cal.

$$\therefore 30(20s + 87{\cdot}2) = 105 \times 27{\cdot}8$$

$$\therefore 20s + 87{\cdot}2 = \frac{105 \times 27{\cdot}8}{30} = 97{\cdot}3$$

$$\therefore s = \frac{97{\cdot}3 - 87{\cdot}2}{20} = \frac{10{\cdot}1}{20} = \mathbf{{\cdot}505}$$

Questions on Chapter VIII

(**1**) Explain the terms: calorie, B.Th.U., specific heat, water equivalent.

A calorimeter weighing 100 grams contained 370 grams of water at 17° C. 130 grams of iron (sp. ht. = 0·11) at 100° C. were dropped into the water, the temperature of which rose to 20° C. Calculate the specific heat of the material of which the calorimeter was made. (O.)

(**2**) What meaning do you attach to the statement that 1 Therm is equal to 100,000 British Thermal Units?

The water for a hot bath is heated by gas and 30 gallons are required at a temperature of 104° F., the temperature of the water supply being 50° F. If the price of gas is 8*d*. per therm, what is the least cost of the heat used?

(1 gallon of water weighs 10 lb.) (L.)

(**3**) A calorimeter, of water equivalent 14 gm., contains 50 gm. of water at 25° C. 160 gm. of water at 46° C. are added. What is the resulting temperature? (O.)

(**4**) What do you understand by the *thermal capacity* of a body?

0·4 gm. of a slow-burning powder is ignited in a closed copper vessel weighing 120 gm. completely immersed in

240 gm. of water contained in a calorimeter of water equivalent 48. If the original temperature is 13° C. and the resulting temperature is 42° C., what is the heat developed on burning 1 gm. of powder?

If you were performing this experiment, what precautions would you take to prevent gain or loss of heat by the calorimeter from the surroundings?

(Specific heat of copper = 0·1.) (O.)

(**5**) Explain what is meant by the *latent heat of fusion*, the *specific heat* of a substance. Do the values of these quantities depend upon the unit of mass or the scale of temperature used?

A metal vessel contains 60 gm. of water at 15° C. When 75 gm. of water at 45° C. are poured into the vessel the temperature changes to 31° C. What is (*a*) the heat capacity, (*b*) the water equivalent of the vessel? What other information must be given, if one wishes to obtain the specific heat of the vessel? (D.)

(**6**) 22 grams of metal (specific heat 0·05) were removed from a furnace and dropped rapidly into 200 grams of oil (specific heat 0·5) at a temperature of 15° C. in a calorimeter of water equivalent 8·9 grams. The temperature of the oil rose 12° C.; what was the temperature of the furnace? (O.)

(**7**) Distinguish between the meaning of *temperature* and *quantity of heat.*

A piece of rock weighing 300 gm. is raised to a temperature of 100° C. and then quickly placed in a copper calorimeter containing 350 gm. of water at 10° C. The calorimeter weighs 250 gm. and the specific heat of copper is 0·1. The final temperature of the mixture after careful stirring is found to be 25° C. Find the specific heat of the rock. State clearly what assumptions you make in carrying out your calculations. (O.)

(**8**) Define the latent heat of fusion of ice.

Describe how you would determine this quantity experimentally. State the possible sources of error which might arise in your method and the means you would take to eliminate them. (L.)

(**9**) Define *specific heat, latent heat of fusion.*

An aluminium calorimeter of mass 50 gm. contains 120 gm. of oil at a temperature of 25° C. Ice is added to the oil, and when it has all melted the temperature is 9° C. The total mass of the calorimeter and contents at the end of the experiment is 183 gm. Calculate the specific heat of the oil. (Latent heat of fusion of ice = 80 calories per gm. Specific heat of aluminium = 0·22.) (C.)

(**10**) Define *specific heat* and *latent heat of fusion.*

Liquid oxygen at its boiling point is contained in a thermos flask. A piece of iron weighing 8 gm. and held by a thread is transferred from the oxygen to a beaker of water at 0° C. A shell of ice weighing 1·63 gm. forms round the iron. Calculate the temperature of the liquid oxygen. The specific heat of the iron may be taken as 0·09 and the latent heat of fusion of ice as 80 calories per gram. (C.)

(**11**) Explain what is meant by the *specific heat* of a substance. Ice at a temperature of — 10° C. is introduced into a copper calorimeter weighing 90 gm. and containing 150 gm. of water at 25° C. The temperature reached when all the ice is melted is 10° C. If the mass of ice added is 25 gm. and the latent heat of fusion of ice is 80 cal./gm., calculate the specific heat of ice.

(Specific heat of copper = 0·1.) (N.)

(**12**) 260 gm. of sulphuric acid at 16° C. are placed in a glass vessel standing in an ice calorimeter, and 33·8 gm. of ice are melted. 100 gm. of water at 16° C. are then added, and a further 140 gm. of ice are melted. Find the specific heat of sulphuric acid, and also the heat evolved on mixing the above quantities of acid and water at 0° C.

(L.H. of fusion of ice = 80 cals. per gm.) (O.)

(**13**) Explain how you would measure the latent heat of water, giving all the precautions you would take to obtain an accurate result.

How much heat is required to convert 10 gm. of ice at — 5° C. into steam at 100° C.?

(Latent heat of water = 80 cals. per gm.
Latent heat of steam = 537 cals. per gm.
Specific heat of ice = ·5 cals. per gm.)
(O. & C.)

(**14**) Describe, in detail, an experiment for determining the latent heat of steam.

Steam at 100° C. is passing into the inner tube of a condenser which is cooled by a constant stream of water flowing at the rate of 1 litre per minute. If the water enters at 20° C., and it and the condensed steam leave at 40° C., how much steam is being condensed per hour? [Latent heat of steam = 540 cal. per gm.] (C.)

(**15**) A calorimeter, of negligible heat capacity, weighs 20 grammes. Water is poured into it, and it is suspended over a bunsen flame by means of a spring balance.

Simultaneous values of the weight and of the temperature are taken and tabulated as follows:

Time (minutes)	0	1	2	3	4	5	6	7	8	9	10
Total weight (grammes)	170	170	170	170	170	164	159	153	147	141	136
Temp. (°C.)	20°	41°	55°	80°	100°	100°	100°	100°	100°	100°	100°

Draw a graph to show the relation between temperature and time. From the data given determine the latent heat of steam, and point out the principal sources of error in this method. (A.)

(**16**) What do you understand by the statement that the latent heat of steam is 540 calories per grm.?

A copper kettle weighs 840 grm. and one litre of water is put into it. Its temperature before being placed on a gas ring is 16° C., and it takes 6 min. for the water to commence to boil. How long would it be before all the water boiled away if heat were still supplied at the same rate? (N.)

(**17**) Explain the statement that the *latent heat of vaporization of alcohol is* 371 *B.Th.U per lb.*, and define the heat unit used.

A copper kettle, weighing 2 lb. and containing 3 lb. of water at 52° F., is placed over a burner, and the water starts to boil after 10 min. Neglecting losses of heat to the surroundings, find what fraction of the heat supplied has been absorbed by the kettle itself.

What further time will elapse before $\frac{1}{2}$ lb. of the water has boiled away?

(Take the specific heat of copper as 0·1, and the latent heat of steam as 972 B.Th.U. per lb.) (N.)

(**18**) Explain what is meant by the statement "the melting point of wax is 55° C., its specific heat is 0·70, and its latent heat of fusion is 35 calories per gramme."

Calculate how much steam from water, boiling at 100° C., will just melt 200 gm. of wax at an initial temperature of 15° C. (O. & C.)

(**19**) Define *specific heat, latent heat of vaporization.*

A piece of copper of mass 2·5 gm. at a temperature of 14° C. is dropped into liquid nitrogen at its boiling-point, — 196° C. The nitrogen evaporated was collected, and was found to have a volume of 712 c.c. at 14° C. and 750 mm. pressure. Calculate the latent heat of vaporization of nitrogen. (Density of nitrogen at N.T.P. = 1·25 gm. per litre; specific heat of copper = 0·08.) (O. & C.)

(**20**) During a test of a boiler, it was found that, for every pound of coal burnt, 10 lb. of water was evaporated at a pressure of 100 lb. per sq. in. (absolute), the feed water being supplied at 128° F. The heating value of the fuel was 12,500 British thermal units per pound; what proportion of the heat of the fuel was being usefully employed? At a pressure of 100 lb. per sq. in., water boils at 328° F., and the latent heat is 883 B.T.U. per lb. You should assume that the specific heat of water remains at unity throughout the range of temperature. (A.)

CHAPTER IX

Change of State

When heat, a form of energy, is supplied to a substance it usually produces a rise of temperature ; that is, it causes an increase in the average kinetic energy of motion of the molecules of which the substance is made up.

It may, however, produce a " change of state " without a change of temperature, by dragging the molecules apart against their forces of attraction or cohesion.

Thus, at a certain temperature, the addition of heat to ice will cause it to melt into the liquid form, water ; at another temperature the liquid may boil and become a vapour or gas.

Melting Point

Determination of Melting Point

Method 1.

A few specks of the substance are placed in a short piece of thin walled capillary tube closed at one end. Fig. 15.

This tube is attached to the bulb of a thermometer by means of a rubber band and immersed in a beaker containing a suitable liquid. (Water to 100° C. ; strong sulphuric acid to 300° C., medicinal paraffin to 200° C.)

The beaker is gradually heated and its contents well stirred until the solid just melts, i.e., it forms transparent drops of liquid.

The temperature or melting point is then read accurately.

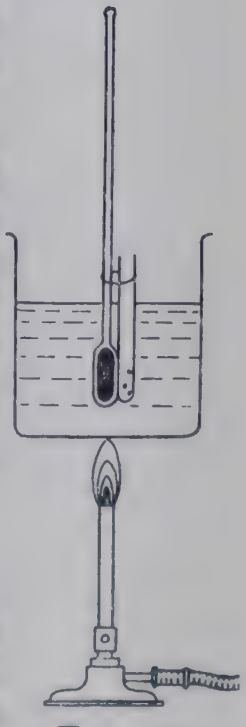


FIG. 15

Method 2.

A thermometer is placed in some of the molten substance contained in a boiling tube. This is allowed to cool, its temperature is observed every half-minute, and the results are plotted on squared paper. Fig. 16.

At the melting point the curve shows a horizontal portion where the latent heat released supplies the heat which is being given out from the hot surface of the tube.

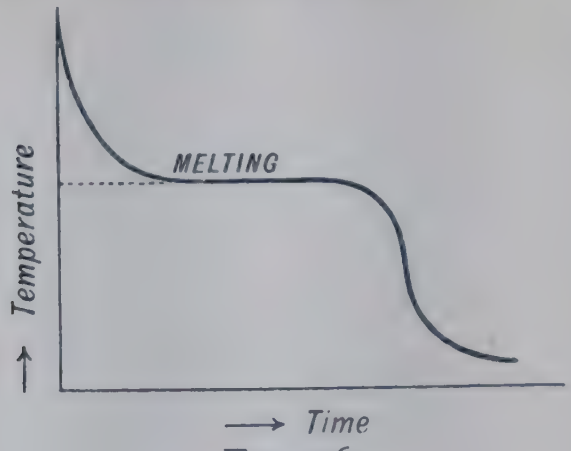


FIG. 16

Effect of Pressure on Melting Point

The melting point of a solid, unless otherwise stated, is assumed to be taken at standard atmospheric pressure. (760 mm. of mercury.)

In the case of substances, like paraffin wax, which expand on melting, the melting point is raised if the pressure is increased.

For ice, which contracts on melting, an increase of pressure lowers the melting point.

This effect may be shown by suspending weights from the ends of a piece of metal wire resting on a block of ice. (See fig. 17.)

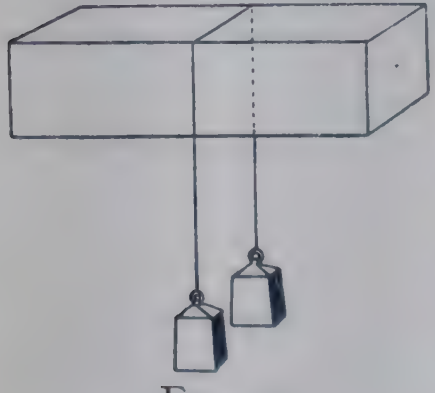


FIG. 17

Immediately below the wire the ice is under pressure, its melting point is lowered and hence at its present temperature, 0° C., it becomes liquid. The wire drops through the thin film of water, which, being now above the wire and not under pressure, becomes ice again.

This phenomenon in which the wire slowly passes through the the ice and yet leaves the block uncut is called "*regelation*."

Similarly two pieces of ice when pressed together form a film of water, which becomes ice and cements the two pieces together when the pressure is released. This accounts for the binding of fresh snow into snow balls.

Vapour Pressure and Evaporation

A and B represent two simple mercury barometers, fig. 18. By means of a short bent glass tube, a drop of a liquid is allowed to float to the top of the mercury in B. In the vacuum the small drop of liquid will completely evaporate and the fall of the mercury measures the pressure of the vapour of the liquid. The space is probably still *unsaturated* and another drop of liquid will increase the effect.

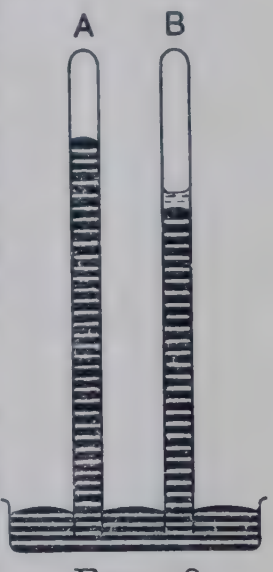


FIG. 18

Eventually a point is reached when some of the added liquid does not vapourise. The space above the mercury is then said to be *saturated*, and the depression of the mercury level below that in A measures the *maximum or saturated vapour pressure* for this liquid at the surrounding temperature.

Effect of Temperature

If the upper part of barometer B is surrounded by a jacket containing water or some suitable liquid whose temperature can be gradually raised, it is found that at higher temperatures a greater vapour pressure may be reached before the space is saturated.

In fact, any liquid has its own saturated vapour pressure which increases as the temperature is raised.

The Boiling Point of a liquid is the temperature at which its maximum Vapour Pressure is equal to that of the external pressure (usually that of the atmosphere) to which it is exposed.

Thus the boiling point is reduced under diminished pressure, and conversely.

This effect may be very considerable. In the case of

water a change of boiling point of 1° C. is produced by a change of pressure of about 28 mm.

N.B. The boiling point of a liquid is also raised by the presence of any non-volatile substance dissolved in it.

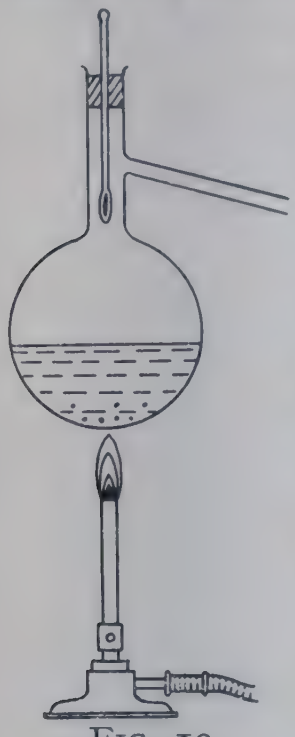


FIG. 19.

Determination of Boiling Point

Method 1 (pure liquid).

The liquid is contained in a small "distillation flask" fitted with a cork and thermometer placed as shown in fig. 19.

The exit tube should be fitted with a condenser to collect the liquid again.

Some small pieces of pipe-clay or broken glass in the liquid will prevent "bumping."

Method 2.

(Convenient when only a few drops of the liquid are available.)

A bent J-tube, with its short limb closed, is filled with mercury, except for a small length of the open limb in which a little of the given liquid is placed.

By placing the thumb on this end and inverting the tube this liquid is transferred to the closed limb. Mercury is then poured out until its level in the open limb is below that in the closed one.

This apparatus is now placed, as in fig. 20, in a flask containing water (or sulphuric acid, etc., if temperatures above 100° C. are required) which can be heated gradually.

The boiling point of the liquid will be reached when its vapour pressure equals atmospheric pressure as indicated by the mercury in the two limbs reaching the same level.

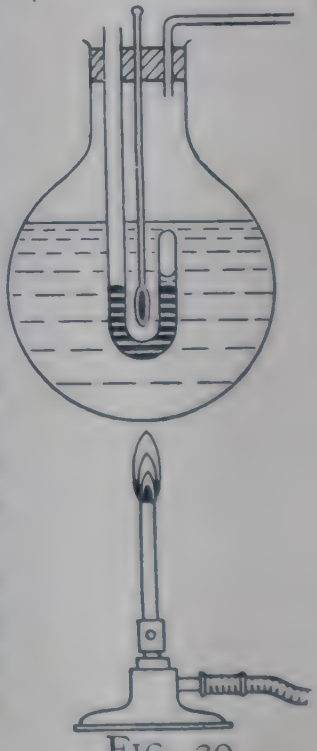


FIG. 20

Volume Changes and Change of State

The curve (fig. 21) shows the volume changes which occur when the temperature of 1 gm. of ice at — 10° C. is gradually raised until the ice becomes steam.

It is not drawn to scale, owing to limitations of space.

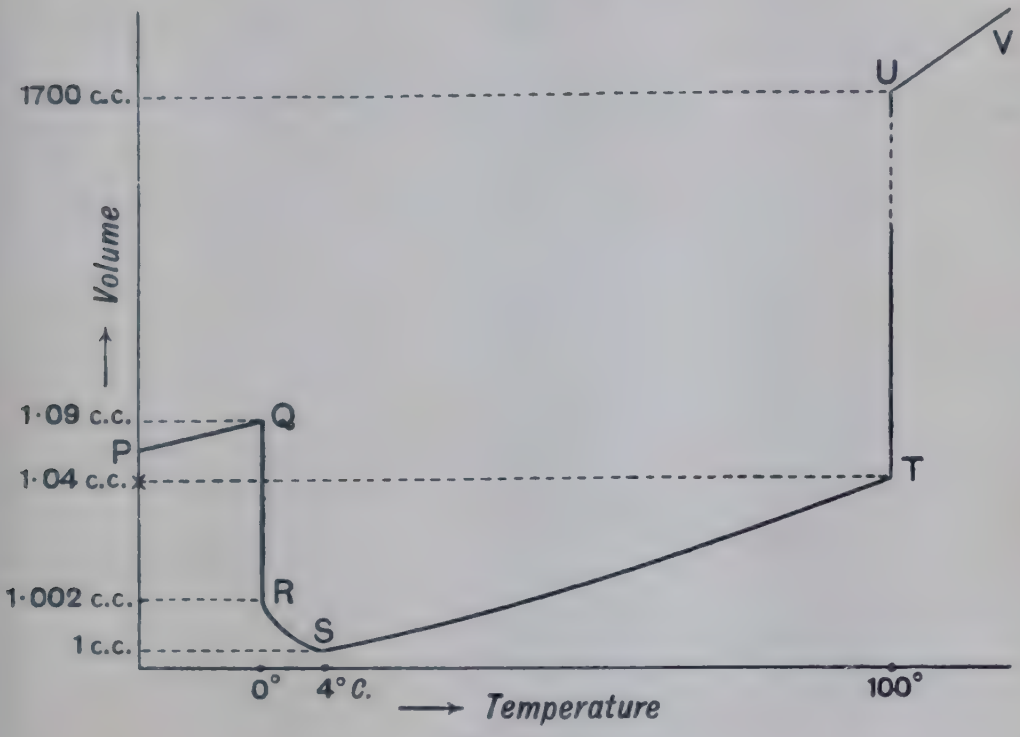


FIG. 21.

PQ. Slight expansion of ice from —10° C. to 0° C.

QR. Contraction on melting at 0° C. 11 c.c. ice become 10 c.c. water.

RS. Small contraction from 0° C. to maximum density of water at 4° C.

ST. Expansion of water from 4° C. to 100° C. coefficient increases with temperature.

TU. Expansion on boiling at 100° C. 1 c.c. water becomes 1700 c.c. steam.

UV. Expansion of steam.

Water Vapour in the Atmosphere—Hygrometry

The *Relative Humidity* of a sample of air is the ratio of the weight of water vapour actually present

to that weight which would be required to saturate it at the given temperature.

Instruments designed to measure the Relative Humidity of air are called *Hygrometers*.

Direct Measurement of Relative Humidity

A measured volume (about 5 litres) of water is allowed to flow out slowly from a large vessel so as to draw an equal volume of air through a series of U-tubes containing calcium chloride or some other drying agent.

The U-tubes are weighed before and after, and the increase in weight gives the weight of water vapour absorbed from a known volume of air.

The experiment is repeated using an equal volume of air which has been saturated by bubbling through water, or through tubes containing wet pieces of broken glass.

Relative Humidity is often expressed as a percentage

$$= \frac{\text{Wt. of water vapour present in a volume of air}}{\text{Wt. of water vapour present in eq. vol. of sat. air}} \times 100$$

Usually the denominator is not obtained by experiment but from standard tables which give the amounts of water vapour required to saturate a known volume of air at different temperatures.

The apparatus used in this experiment is sometimes called the chemical or weight hygrometer.

Dew Point

The Dew Point is the temperature at which dew is deposited, that is the temperature at which the amount of water vapour present in the air, at any moment, is just sufficient to saturate it.

Indirect measurement of Relative Humidity.

The relative humidity can be obtained when the dew point has been observed because it is found that (to within one or two per cent.) :—

$$\frac{\text{Wt. of water vapour present in air}}{\text{Wt. of water vapour which would saturate the air}} = \frac{\text{Saturated Vapour Pressure of water at Dew Point}}{\text{Saturated Vapour Pressure of water at air temperature}}$$

Tables have been compiled showing the saturated vapour pressures of water at various temperatures. The values of the saturated vapour pressures can be written down from these tables.

Thus at 20° C. the Saturated Vapour Pressure of water is 17·5 mm. of mercury. Also at the dew point (found on a certain day to be 12° C.) the S.V.P. is 10·5 mm.

$$\text{Hence Relative Humidity} = \frac{10{\cdot}5}{17{\cdot}5} = 0{\cdot}60 = 60\%.$$

Regnault's Hygrometer. Fig. 22.

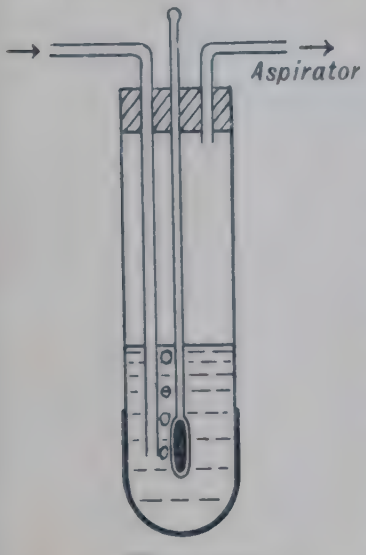


FIG. 22.

Some ether is contained in a glass tube, the lower end of which is covered with polished silver. A slow stream of air is drawn through the ether, causing evaporation and hence a gradually falling temperature. A film of dew eventually forms on the surface of the silver, and the thermometer in the ether registers the Dew Point. A more accurate value is obtained by taking the average between the temperatures at which dew first appears and later disappears.

Dew forms on grass, trees, railings, etc., when the temperature of the earth's surface falls sufficiently below that of the air to reach the Dew Point.

This occurs most frequently on clear still nights when radiation of heat from the earth is rapid, and evaporation slow.

Wet and Dry Bulb Hygrometer

This consists of two thermometers hanging side by side. The bulb of one is kept wet as it is surrounded by muslin or wick which dips into a small glass of water.

If the air is not already saturated, then water will evaporate from the wet bulb causing a fall in its temperature.

Thus the difference in temperature between the two thermometers will depend on the Relative Humidity or

Hygrometric State of the atmosphere. The connection is complicated but the required fraction can be obtained by reference to tables. The simplest form of these shows the hygrometric state of the air tabulated against air temperatures and the differences of temperature between the wet and dry bulbs.

Questions on Chapter IX

(**1**) Explain clearly why, when heat is supplied to melting ice, the temperature does not rise until all the ice has melted. What becomes of the heat energy so supplied?

How much steam at 100° C. must be passed into a mixture of 50 gm. of ice and 100 gm. of water contained in a calorimeter of mass 100 gm. and specific heat 0·1, so that the final temperature of the mixture shall be 10° C.?

(Latent heat of water = 80 calories per gm.
Latent heat of steam = 536 calories per gm.) (O. & C.)

(**2**) Explain exactly what is meant by the statement that "the maximum vapour pressure of water at 30° C. is 3·171 cm."

Describe an experiment which would enable you to verify this statement.

Sketch a graph showing in a general way how the vapour pressure changes between 0° C. and 100° C. (B.)

(**3**) What is meant by the "boiling point" of a liquid? Explain clearly the difference between the processes of evaporation and boiling.

A beaker containing 240 grams of water at 12° C. was placed over a lighted Bunsen burner and five minutes later the water was boiling. After another ten minutes had elapsed it was found that 80 grams of water had boiled away. Calculate the latent heat of steam. (D.)

(**4**) Describe an experiment to show that the vapour pressure of ether is greater than that of water at room temperature, and another to show that the vapour pressure of water boiling in the open air is equal to the pressure of the atmosphere. (N.)

(**5**) Heat is supplied at a regular rate to a quantity of ice at 0° C. finally changing it to steam at 100° C. Illus-

trate by graphs, pointing out briefly their distinguishing features, (*a*) how the temperature varies with time, (*b*) how the volume of the water in its different physical states varies with temperature? (L.)

(**6**) Explain the formation of dew. Why does it form most readily on calm, clear nights?

Assuming one millimetre to be the average depth of deposition of moisture in the form of dew, calculate (*a*) the quantity over a square metre of ground, and (*b*) the corresponding amount of heat liberated during the condensation, supposing the dew point remained at 5° C., at which point the latent heat of vaporization is 593 calories per gram. (A.)

(**7**) Describe **two** methods by which you could determine accurately the relative humidity of the atmosphere. What observations would you make to obtain a rough estimate of the humidity of the atmosphere? (O.)

(**8**) Define dew-point, and describe an experiment by which it may be determined.

Explain what is meant by the relative humidity of the air. Why does an east wind in England usually feel colder than a south-west wind of the same temperature? (O.)

CHAPTER X

The Transmission of Heat

Conduction is the flow of heat, through an unequally heated body, from places of higher to places of lower temperature.

There is no perceptible motion of the parts but these are all raised in temperature as the heat passes through them.

In terms of *molecules*, we suppose that the molecules at any point have a certain average kinetic energy of vibration, and, by collision, transfer some of this energy to adjacent particles and so on.

This sharing of heat energy will mean a fall in average velocity and therefore temperature. Further, in the case of, say, a bar of metal, some energy will also be radiated from the surface (see radiation below).

In general metals are good conductors of heat, non-metals are poor conductors.

Experiment to show that some metals are better conductors than others

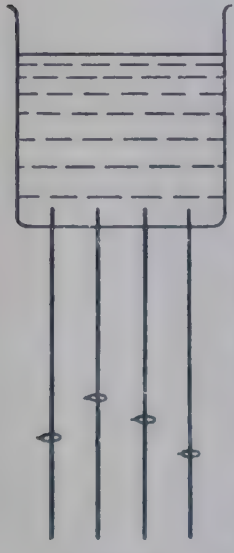


FIG. 23

A series of rods of different metals having the same length and cross-section are fixed into the bottom of a metal can as shown in fig. 23.

The rods are coated with a thin layer of wax which holds small metal rings in position. Hot water is poured into the can; heat is conducted along the rods and as any point on any rod reaches the temperature of the melting point of wax, the rings slide down.

When the rings finally come to rest (at points of equal temperature), their positions will show some metals to be better conductors than others.

It can be proved that the thermal conductivities are proportional to the squares of the distances travelled by the rings.

The effect of specific heat

It is important to notice that the above lengths must be measured only when a "*steady state*" of temperature has been reached and the rings are stationary.

In the above experiment it may be that a certain ring will start moving downwards more quickly than another ring, but that the latter will have fallen a considerably greater distance eventually.

The explanation is that although more heat may pass along the second rod, owing to its higher *conductivity*, yet this heat may not produce as high a temperature, at a point near the source of heat as at a corresponding point in the first rod if the material of this first rod has a much lower heat capacity per unit volume.

For example iron has a conductivity about twice that of lead; its specific heat is about three times and its density is about two-thirds that of lead.

Thus the ring on the iron rod will move less rapidly than that on the lead rod, at first, but will eventually, when the steady state is reached, have moved four times as far; ($(\text{distance})^2 \propto$ conductivity—see above).

Effects of Conduction

Liquids, except for liquid metals such as mercury, are poor conductors.

For example water may be boiled by carefully applying a bunsen flame to the top of a test tube filled with water whilst the bottom is held in the hand.

Convection (see below) is thus eliminated, and any heat reaching the lower layers of the water must do so by conduction through the water or the glass.

Gases

Gases also are bad conductors, but in making experiments it is difficult to eliminate convection effects.

Blankets may be used to keep a warm body warm, but will also keep a cold body cold, e.g., a piece of ice.

The poor conductivity of loosely woven clothing is due to the air in the interspaces.

For similar reasons house roofs lined with felt are a screen against both heat and cold.

Solids

Saucepans, kettles, boilers, etc., are made of materials which are good conductors.

A clean wire gauze can " push down " the flame of a bunsen burner. Moreover if the gauze is held over an unlighted bunsen, the gas can be lighted and allowed to burn only above the gauze.

This is due to the fact that the strands of wire conduct the heat quickly away to the edges before the space on the other side of the gauze reaches the temperature necessary to ignite the gas-air mixture.

For this reason a screen of gauze is effective in the Davy miner's safety lamp.

Thermal Conductivity as a Quantity

The Thermal Conductivity of a substance is defined as the quantity of heat (in calories) which can pass, per second, through a section of area 1 sq. cm. of the substance, when the fall of temperature is 1°C. per 1 cm. of length in the direction in which the heat is passing.

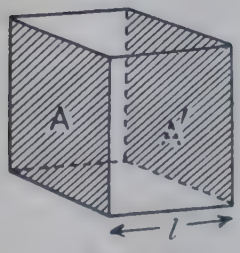


FIG. 24

Thus if fig. 24 represents a thin section of material of cross-sectional area A sq. cm. and length l cms., and if the temperature difference between A and A^1 is $\theta°$ C. then the heat passing from A to A^1 in time t secs. is given by the expression

$$Q = k \times \frac{A \times \theta \times t}{l}$$

where k = a constant, the thermal conductivity of the material.

Convection

Convection is the transference of heat by the motion of the hot body itself carrying its heat with it.

This occurs only in liquids and gases and the motion is usually produced by variations in the density, with temperature, of different parts of the fluid.

Effects due to Convection

Winds

The air above the earth's surface near the equator is hotter than elsewhere and therefore rises. Cooler air therefore moves, along the earth's surface, to take its place from the north and from the south. Owing to the rotation of the earth these " winds " appear to blow from the N.E. and S.E. respectively constituting the Trade Winds.

Near the coast, the land, during the day time, becomes hotter than the sea. Hot air therefore rises above the land and a cool sea breeze blows to take its place. At night the land cools more rapidly than the sea and a land breeze occurs.

Ventilation

A fire in a room assists ventilation, for a column of hot gases is continuously rising up the chimney, and fresh air will enter the room, through the doors and windows, to take its place.

Better ventilation will be obtained in a warm room if a window is open partly at the top and partly at the bottom, than if it is only open at the top or bottom.

For in this way the warm air escapes at the top and fresh air enters at the bottom.

Convection in Liquids

The movement of the particles is more readily seen in liquids than in gases.

For example if a crystal of potassium permanganate, which dissolves slowly in water giving a deep purple solution, is placed in a flask of water and warmed, convection currents will be seen as shown in fig. 25.

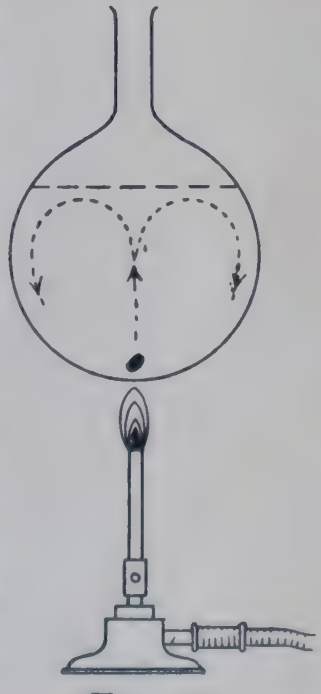


FIG. 25

Ocean currents are to some extent due to convection in the water but mainly to atmospheric convection currents.

Domestic Hot Water Supply

Fig. 26 is a diagram of apparatus for making use of convection currents to give an efficient hot water supply.

B = Boiler with safety valve V.

The water in the boiler only circulates through a nearby cylinder C (Calorifier) and transfers its heat to the water in a surrounding cylinder D, which is fed from the cold water supply tank S.

N.B. This eliminates trouble due to fresh water continuously entering the boiler and depositing its hardness.

The hot water pipe rises vertically from D supplying radiators, baths, etc., on the various floors ending as an overflow pipe to the cold cistern S, to allow for expansion.

As far as possible separate return pipes lead back to D.

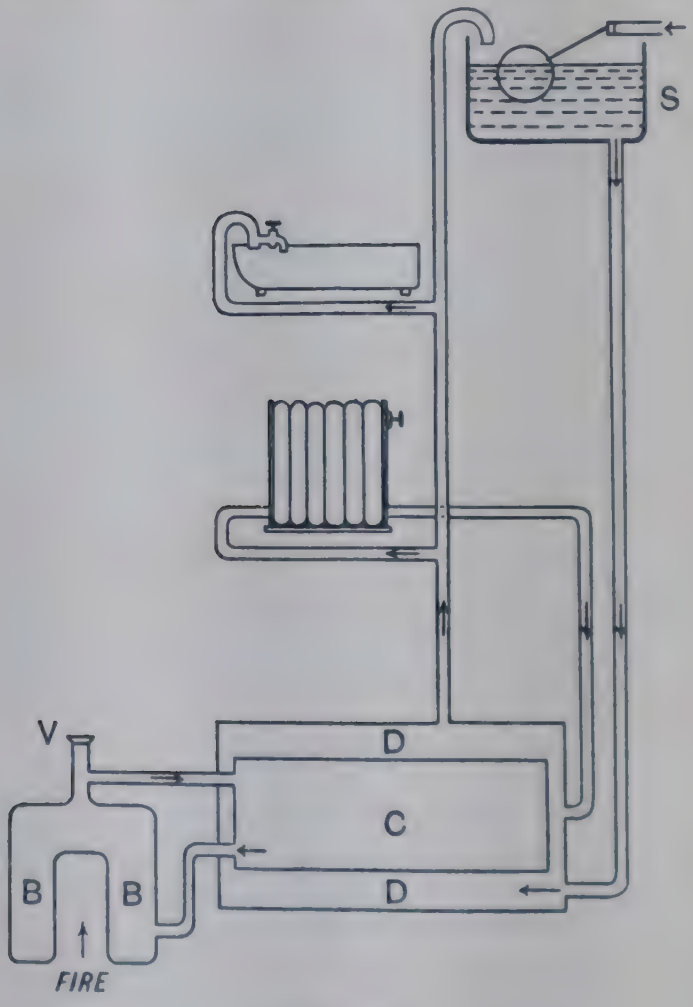


FIG. 26.

The principle can be shown experimentally with the simplified apparatus, fig. 27, if the water in the upper glass vessel is coloured with potassium permanganate before the heat is applied to the lower flask.

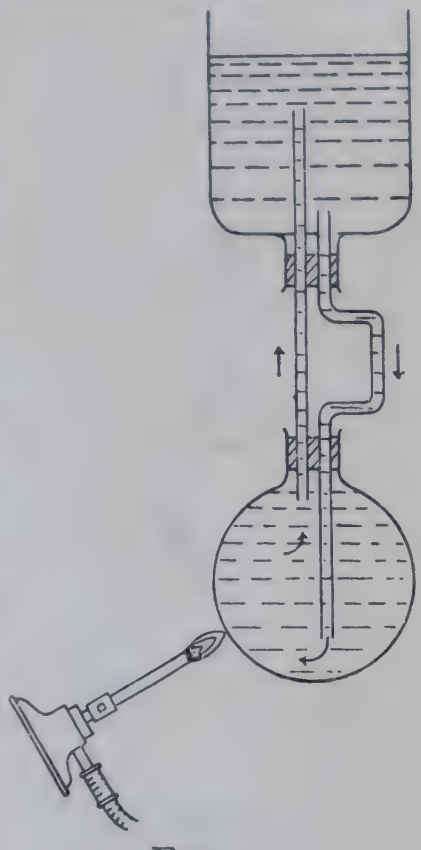


FIG. 27

Radiation

In Radiation the hotter body loses heat and the colder body receives heat by means of a process occurring in some intervening medium which does not itself thereby become heated.

Radiant Heat Energy is supposed to be transmitted by transverse waves in the ether. These travel with the same velocity as Light Waves, but are of longer "Wave Length."

Experiments can be devised to show that these waves—

(*a*) Travel in straight lines.
(*b*) May be reflected and refracted like light waves.
(*c*) Pass through a vacuum.

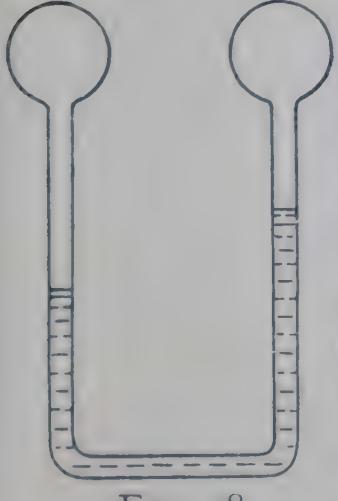


FIG. 28

Radiant Heat is invisible but may be detected on an ordinary thermometer or better by an air thermometer with blackened bulb, or by a Thermocouple or a Thermopile.

Differential Air Thermometer. Fig. 28.

Any difference in temperature between the two blackened bulbs, which contain air, is indicated by a difference in level of the liquid in the two limbs.

Thermocouple and Thermopile

If strips of two different metals are joined together at their ends (fig. 29*a*),

and if one junction A is hot and the other, B, is cold, then a small electric current will pass round the circuit and can be detected by a sensitive galvanometer G. The metals antimony and bismuth show this effect particularly well.

This effect may be multiplied by arranging many strips of the metals to form a cube—a thermopile, figs. 29*b* and 29*c*. One face of the cube is covered with a metal cap and the opposite face is fitted with a metal cone whose inside surface is silvered and receives the radiant heat.

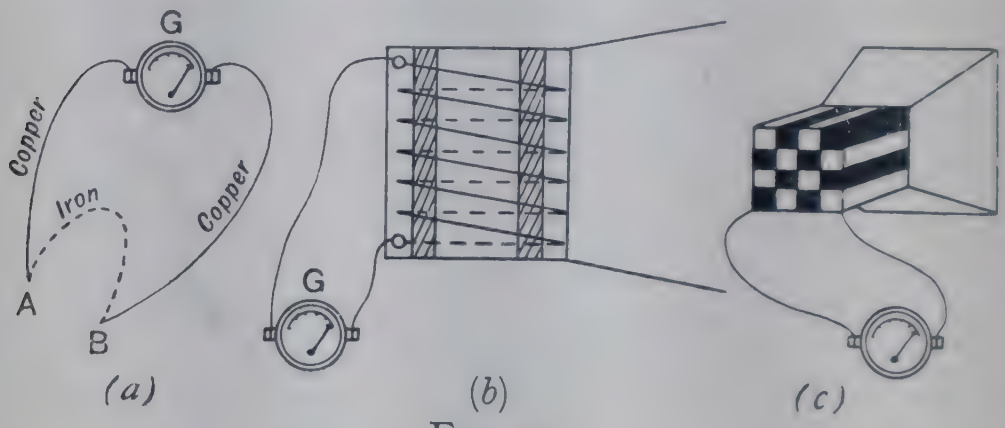


FIG. 29

To show that the amount of heat radiated by a hot surface depends on the nature of the surface.

A tin can in the shape of a cube has one face brightly polished, another covered with lamp black, another rough and dull, and another painted with aluminium paint. This is called a " Leslie's Cube."

Such a cube is filled with hot water and placed midway between the two bulbs of a differential air thermometer. (Both bulbs should be blackened, see below.) By turning the cube it can be shown that the lamp-black or dull surface is a better radiator than the polished surface, even though all surfaces are at the same temperature.

To show that the heat absorbed by a surface depends on the nature of the surface.

The above experiment is repeated using a Leslie Cube with two exactly similar surfaces, and a differential air thermometer of which one bulb is blackened and the other silvered.

The liquid on the blackened bulb side of the thermometer immediately falls. A thermocouple may be used instead of the air thermometer in these experiments.

In general it can be shown that surfaces which radiate well, when hot, also absorb well, when cold.

Effects due to Radiation

1. Heat reaches the Earth from the Sun without raising the temperature of the upper atmosphere. This radiant heat is then absorbed by the Earth's surface producing a rise in temperature.

2. Teapots and kettles have brightly polished surfaces partly for aesthetic reasons and partly because they thus lose the minimum of heat by radiation.

3. Dark rough clothing absorbs more radiant heat than white clothing in sunny weather.

4. *Thermos Flasks*

The vacuum between the inner and outer cylinders prevents loss of heat by convection and conduction. Loss by radiation is reduced to a minimum by having the outside of the inner cylinder and the inside of the outer cylinder polished.

A vacuum flask will thus keep hot substances hot, or cold substances cold.

Diathermancy

Substances are said to be *diathermanous* when they allow heat waves to pass through them, just as some substances are said to be *transparent* to light.

Adiathermanous corresponds to *opaque*.

As with Light so with Heat, the " transparency " of a substance will vary with the wavelength of the incident radiation.

Greenhouses

A very hot body such as the Sun gives a large proportion of its heat in the form of waves of relatively short wave length. These pass through glass and are absorbed by the soil and walls, etc., inside the greenhouse. The soil

and walls becoming warm, give off radiation of much longer wave length to which the glass is less transparent. A large proportion of heat therefore remains inside the greenhouse.

The Rate of Cooling of a Hot Body

The quantity of heat lost in a given time, by a body, due to radiation and convection in the air depends on :—

(*a*) The temperature of the body relative to that of its surroundings.

(*b*) The nature of its surface.

(*c*) The area of the surface.

For a given type and area of surface, and neglecting conduction effects, **Newton's Law of Cooling states that the quantity of heat lost per second by a body is proportional to the difference in temperature between the body and its surroundings.** This is approximately true for any warm body hung in the air so that it loses heat mainly by radiation and convection. It is not true for a body whose temperature is very much greater than that of its surroundings.

Specific Heat by the Method of Cooling

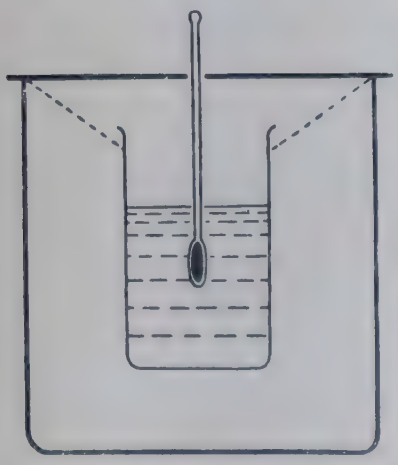


FIG. 30.

A small metal vessel with its outside surface blackened is supported by threads, or on non-conducting supports, inside a larger vessel whose inside surface is blackened (fig. 30). As a further refinement the outer vessel may be kept at constant temperature by a jacket of ice-cold water.

The inner vessel, previously weighed, is filled to about $\frac{2}{3}$rds of its volume with hot water, and the time (t_1 secs.) taken for its temperature to fall from say 50° C. to 40° C. is noted.

A further weighing gives the weight of water, M_1 gm.

The experiment is repeated using *an equal volume* of some other liquid.

The time taken for the temperature to fall from 50° C. to 40° C. is noted = t_2 secs.

The weight of this liquid is determined. = M_2 gm.

Calculation

Let w = water equivalent of the calorimeter.

Let s = specific heat of the liquid.

The average excess temperature above the surroundings is the same in each case, hence the rate of loss of heat is the same.

$$\therefore \frac{(M_1 + w) \times 10^\circ}{t_1} = \frac{\{(M_2 \times s) + w\} \times 10^\circ}{t_2}$$

$$\therefore M_2 s + w = \frac{t_2}{t_1}(M_1 + w)$$

s can then be calculated.

Questions on Chapter X

(1) Explain what is meant by the statement "the thermal conductivity of glass is 0·002 in C.G.S. units." The inside of a glass window 2 mm. thick and 1 sq. m. in area is at a temperature of 15° C. and the outside is at — 5° C. Calculate the rate at which heat is escaping from the room by conduction through the glass. (O. & C.)

(2) One end of a bar of metal, A, just passes through a cork which fits in a hole in the side of a vessel which is provided with inlet and outlet tubes. At a given instant a rapid stream of steam is admitted to the vessel and at the end of two minutes the temperature of the bar at a point 6 inches from the heated end has risen 15° C. The experiment is then repeated with a bar of a different metal, B, whose dimensions are the same as those of the first bar, and it is found that the time required for the same rise of temperature to occur is three minutes. Explain why the ratio of the conductivities of the two metals is not that of 2 to 3.

A pool is covered with a sheet of ice, 2 cm. thick, and the temperature of its upper surface is — 2° C. Calculate approximately how much the ice will thicken in one hour. Latent heat of ice = 80. Conductivity of ice 0·006. (A.)

(**3**) Distinguish between *conduction* and *convection* of heat, giving one example of each process.

Explain clearly what would be a suitable position for a pail of ice, introduced into a room, in order to cool the air in it. (N.)

(**4**) What do you understand by the propagation of heat by convection currents? Illustrate your answer by reference to experiments which you have done or seen.

Making a drawing to illustrate the heating of a house by means of a hot-water circulatory system. (N.)

(**5**) Describe how you would demonstrate in the laboratory—

(*a*) that copper conducts heat better than iron does;

(*b*) that a cold metal gauze prevents the passage of the flame of a burning mixture of gas and air.

Describe the construction of, and explain the working of a piece of apparatus which makes use of (*b*). (O.)

(**6**) A flask of hot water stands on a table. Give a concise account of the various ways in which it loses heat.

In what way would the rate of loss be affected if a large sheet of copper were placed under the flask and a glass beaker inverted over it so as to enclose it completely? (B.)

(**7**) How would you compare experimentally the radiating powers of different surfaces? What steps would you take to make the radiating power of a surface (*a*) as small as possible, (*b*) as large as possible?

Describe the Dewar (*i.e.*, thermos) flask and explain the principles on which its action depends. (D.)

(**8**) Two similar copper calorimeters, each of water equivalent 10 gm., contain 300 c.c. of water and 300 c.c. of alcohol respectively at 50° C. The calorimeters are supported in air so that they may cool under similar conditions. The former cools to 45° C. in 6 min. and the latter in 4 min. What is the specific heat of alcohol, if the specific gravity of alcohol is taken as 0·9? (D.)

(**9**) Instead of heating workshops by means of radiators placed at the level of the floor, it is now common to find them warmed by hot water pipes suspended from the beams

on the roof. Briefly discuss the merits of the latter method, both as to the heating effect and as to hygiene. (A.)

* (**10**) Explain the following :

(*a*) Drinking glasses which are to be used for hot liquids should be as thin as possible.

(*b*) An exposed patch of snow which is covered with soot will melt more quickly than a similar patch which has not been so covered. (N.)

(**11**) Explain the following :

(*a*) An open fire is a good aid to the ventilation of a room.

(*b*) A glass stopper which has become fixed in the neck of a bottle can often be released by warming the neck gently.

(*c*) Steam causes more severe burns than water at the same temperature.

(*d*) The dentist warms his small mirror before putting it in the patient's mouth. (W.)

CHAPTER XI

The Mechanical Equivalent of Heat

Joule in 1843 proved that a definite amount of mechanical energy if allowed to produce nothing but heat always produced a fixed amount of heat.

This constant ratio $\frac{\text{Work expended}}{\text{Heat produced}}$ is known as the mechanical equivalent of heat.

To determine this constant Joule allowed weights to fall a known distance (doing a known amount of work) so as to rotate a paddle wheel in a specially designed calorimeter containing water. The rise in temperature enabled him to calculate the amount of heat produced. The student should learn the details of this or of some other experiment by which the mechanical equivalent of heat can be determined. Various methods have been used including several in which the work expended is supplied and measured in electrical units.

It can be shown that **778 foot-pounds of work are equivalent to 1 B.Th.U.**, i.e., will raise 1 lb. of water 1° F.

In other units **$4{\cdot}27 \times 10^4$ gm. cm. or $4{\cdot}19 \times 10^7$ ergs of work are equivalent to 1 calorie,** i.e., will raise 1 gm. of water 1° C.

In this connection the following definitions and units should be noted :

Mass

The amount of matter which a body contains—measured in gm. or in lb.

Weight

The pull, due to attraction, of the Earth on a body. This will vary with the position of the body on the Earth's surface, being slightly less at the Equator than at the Poles.

Units of Weight

1. *A gram weight* is the Earth's pull on a mass of one gram, at any particular place.

2. *A pound weight* is the Earth's pull on a mass of one pound, at any particular place.

These are really *forces* and Newton showed that a *force* can be measured by the acceleration it will produce in a known mass.

Units of Force can therefore be defined by the accelerations they produce.

1. *A dyne* is the force necessary to give a mass of 1 gm. an acceleration of 1 cm. per sec. per sec.

2. *A gram weight* (see above) is therefore 981 dynes, at a place where the acceleration with which any body falls towards the earth is 981 cm. per sec. per sec.

3. A *poundal* is the force necessary to give a mass of 1 pound an acceleration of 1 ft. per sec. per sec.

4. *A pound weight* (see above) is therefore 32 poundals, at a place where 32 ft. per sec. per sec. is the acceleration due to gravity.

Work

A force does *work* when its point of application moves in the direction of the force, and the work done is the product of force and distance.

Units of Work

1. *An Erg* is the work done when 1 dyne moves through 1 cm.

2. *A Centimetre Gram Weight* is the work done when 1 gm. weight moves through 1 cm.

3. *A Foot-Poundal* is the work done when 1 poundal moves 1 foot.

4. *A Foot Pound weight* is the work done when 1 pound weight moves 1 foot.

5. *A Joule* is defined as 10^7 ergs. It is actually the work done when 1 coulomb of electricity (i.e., 1 amp. for 1 sec.) falls through a potential difference of 1 volt. (See text-books of Electricity.)

Energy is the capacity for doing work.

Units of Energy. As for Work.

Power is the rate of doing work or the rate at which energy is expended.

Units of Power

1. A Horse-Power (H.P.) is 33,000 ft.lb. per minute.
or 550 ft.lb. per second.
2. A Watt is one Joule per sec., i.e., 10^7 ergs per sec.
By calculation 1 H.P. = 746 Watts.

Worked Examples

1. A tube 6 feet long containing a little mercury and closed at both ends is rapidly inverted fifty times. What is the maximum rise in temperature of the mercury that can be expected?
(Specific heat of mercury = 1/30. One B.Th.U. is equivalent to 778 foot-pounds.) (L.)

Let there be W lb. of mercury in the tube.
Work expended each time tube is inverted = $6 \times W$ ft. lb.
$\therefore$ Total Work expended = $50 \times 6 \times W$ ft. lb.
Let $\theta°$ F. = Max. rise in temperature.

Then heat required = $W \times \theta \times \frac{1}{30}$ B.Th.U.

$= W \times \theta \times \frac{1}{30} \times 778$ ft.lb.

Hence $W \times \theta \times \frac{1}{30} \times 778 = 50 \times 6 \times W$

$$\therefore \theta = \frac{50 \times 6 \times 30}{778} = \mathbf{11{\cdot}6°\ F.}$$

2. The energy due to a mass of 5 kilograms falling through a height of 30 metres is used to stir 2 kilograms of water by means of paddles. If all the energy remains as heat in the water, what will be its rise in temperature?
[Mechanical Equivalent of Heat = $4{\cdot}2 \times 10^7$ ergs per calorie; g = 981 cm. per sec. per sec.] (J.M.B.)

5 kilograms falling 30 metres give 5000×3000 gm. cm.
$= 5000 \times 3000 \times 981$ ergs.
Heat Equivalent of this

$$= \frac{5000 \times 3000 \times 981}{4{\cdot}2 \times 10^7} \text{ cals.} = \frac{15 \times 98{\cdot}1}{4{\cdot}2} \text{ cals.}$$

2 kgm. water rising $\theta°$ C. require

$$2000 \times \theta \text{ cals.} = \frac{15 \times 98{\cdot}1}{4{\cdot}2} \text{ cals.}$$

$$\therefore \quad \theta = \frac{15 \times 98{\cdot}1}{4{\cdot}2 \times 2000} = {\cdot}175° \text{ C.}$$

Questions on Chapter XI

(1) What is meant by the mechanical equivalent of heat, and how has it been determined?

Assuming its value to be $4{\cdot}2 \times 10^7$ ergs per gram calorie, find the difference in temperature of the water above and below a waterfall of height 100 metres. (O. & C.)

(2) Explain the statement, $4{\cdot}2 \times 10^7$ ergs are equivalent to one calorie.

From what height must mercury be allowed to fall on to a non-conducting surface in order that its temperature may be raised 1° C.?

(Specific Heat of mercury = 0·033, Acceleration due to gravity = 980 cm. per sec. per sec.) (L.)

(3) A vessel containing liquid at its boiling point, and suitably jacketed to prevent heat losses, is stirred by a motor-driven stirrer using $\frac{1}{10}$ horse-power. If 250 gm. are evaporated in 1 hour, what is the latent heat of vaporization of the liquid?

(1 horse-power = $7{\cdot}5 \times 10^9$ ergs per sec.
J = $4{\cdot}2 \times 10^7$ ergs per calorie.) (N.)

(4) How has it been shown that when work is converted into heat there is a constant ratio between the work done and the heat produced?

In a rough determination of the mechanical equivalent of heat a quantity of lead shot was placed in a cardboard cylinder of length 1 metre, and was allowed to fall backwards and forwards from one end of the tube to the other 50 times in succession. The temperature of the shot was found to have risen 3·7° C. Calculate the mechanical equivalent of heat. (Specific heat of lead = 0·03.) (O. & C.)

(**5**) What is meant by the statement that the Mechanical Equivalent of Heat is 778 ft.lbs. per British Thermal Unit ?

A calorimeter of copper weighed 0·25 lb. It contained 4 lbs. of aniline oil. The liquid was stirred by a rotating paddle, and after 450 revolutions the temperature was found to have risen 14° F. How much work was done on the paddle if all the work was entirely converted into heat ?

(Specific heat of copper = 0·1.
Specific heat of aniline oil = 0·5.) (D.)

(**6**) In order to determine the mechanical equivalent of heat, 690 gm. of mercury was placed in an iron pot weighing 215 gm. mounted on a non-conducting base. On rotating the pot through 400 turns within a loosely-fitting sleeve of felt, the temperature rose through 7·7 Centigrade degrees. The felt sleeve did not rotate, since it was fixed within an outer cylinder of metal, of external diameter 5·4 inches, which was pulled tangentially by a horizontal cord carrying a 2 lb. weight over a smooth pulley, so that the torque due to friction was balanced. Calculate (*a*) the heat, in calories, (*b*) the work, in foot-pounds, and (*c*) the mechanical equivalent in foot-pounds per calorie. (A.)

(Specific heat of iron = 0·113 ; of mercury = 0·0333.)

HEAT

Heat Revision Test (I)

Temperature

The two common scales of temperature are called the (**1**) and the (**2**) scales. They use the same two fixed temperatures, (**3**) and (**4**) and on the first scale these temperatures are called (**5**) and (**6**), whereas on the second scale they are taken as (**7**) and (**8**). To test whether a mercury thermometer gives correct temperatures, it is usual to test the lower fixed point by the apparatus shown in diagram (**9**) and the upper fixed point by (**10**). An alcohol thermometer is usually tested by (**11**).

The advantages of using mercury as the liquid in thermometers are (**12**). Alcohol thermometers have advantages (**13**) and disadvantages (**14**). For measuring the temperatures of liquid gases a (**15**) is used. This instrument is also used to measure the temperatures of molten metals and furnaces. To record the maximum and minimum temperatures in greenhouses a Six's thermometer is generally used. Its action is illustrated in (**16**). A clinical thermometer is shown in (**17**).

It is sometimes necessary to convert a temperature measured in one scale to another scale. Thus 140° C. becomes (**18**) and — 5° C. is (**19**), while 68° F. is (**20**) and 14° F. is (**21**).

Expansion

Most substances expand when heated. The coefficient of linear expansion of a solid is defined as (**22**). Its value for brass is ·000018 per degree C. and therefore it will be (**23**) per degree F. Hence a 10 ft. rod of brass tubing raised from 62° F. to the normal boiling point of water will expand by (**24**).

An apparatus for measuring this coefficient for an aluminium rod is (**25**). A general formula connecting length, temperature and the coefficient of expansion with the length at 0° C. is (**26**).

A pendulum rod expands as the temperature rises so that a clock with an uncompensated pendulum then (**27**). A compensated pendulum is shown (**28**).

The coefficient of volume expansion of a solid is (**29**) times the coefficient of linear expansion. A proof of this statement is given (**30**).

Liquids have to be enclosed within containers which themselves expand. Hence the apparent expansion of the liquid as measured in a glass vessel is (**31**) than the real expansion. For mercury the real coefficient is ·000181 per degree C. Therefore if the *linear* expansion of the glass vessel is ·000010, the apparent expansion of the mercury will be (**32**).

The apparent expansion of a liquid can easily be measured by an experiment with a specific gravity bottle. The procedure is (**33**) and the coefficient of apparent expansion is given by (**34**).

A direct measurement of absolute expansion can be made by balancing a hot against a cold column of liquid as in figure (**35**). The coefficient of absolute expansion is then given by the equation (**36**).

The changes in volume of 1 gm. of ice-water when it is heated from — 10° C. to 10° C. are illustrated by the graph (**37**).

Gases differ from solids and liquids in that they have all nearly the same coefficient of volume expansion which is (**38**) of their volume at (**39**) for 1° C. It is possible therefore to talk of a gas scale of temperature and on this scale there is an absolute zero of temperature. This zero is (**40**)° C. or (**41**)° F. A standard gas thermometer is the ultimate standard by which the accuracy of mercury and other thermometers is determined. The gas equation (**42**) expresses the general behaviour of gases. A gas whose changes of volume exactly agree with this equation is defined as a perfect gas.

Rapid Revision Test (2)

Quantities of heat are usually measured either in calories or in British Thermal Units. The definition of the calorie is (**1**) and that of the B.Th.U. is (**2**). Large quan-

tities of heat energy are measured in Therms where 1 therm = 100,000 B.Th.U. The number of heat units which will raise the temperature of a body one degree is called its thermal capacity. This heat capacity is therefore expressed in calories or in B.Th.U.s.

It is however usually more convenient to think of the specific heat of a material and this is defined as (**3**). The heat capacity of the body is then obtained by (**4**). The quantity of heat which will raise m gm. of a substance of sp. ht. s through $x°$ C. is (**5**). The water equivalent of an object is the mass of water which has the same heat capacity as the object. Its value for mass m and sp. ht. s is (**6**).

In measuring specific heat it is usual to mix the substance with water. The equation which gives the sp. ht. is (**7**). If the substance is a liquid this method will often do but it is sometimes convenient to use the method of cooling described in (**8**).

The Latent Heat of Fusion of a substance is defined as (**9**), and the Latent Heat of Vaporization in a corresponding way. It is therefore necessary to specify both the heat units and the mass units used. For water the values of these two latents heats are approximately (**10**) and (**11**). The apparatus used to measure the latent heat of vaporization of a liquid is shown (**12**), and the equation (**13**) enables the calculation to be made. The energy used to produce a change of state varies if the temperature at which the change occurs is altered.

Some substances expand and others contract when they solidify. It is easy to see to which class a particular substance belongs since the solid will float on the liquid if (**14**) occurs. A substance which contracts as it solidifies is (**15**). For water 1 c.c. of liquid forms (**16**) of ice. For this reason water pipes often break during a (**17**). Great pressure applied to ice causes its melting point to (**18**). The melting point of a solid at atmospheric pressure is found by using the apparatus (**19**).

Evaporation differs from boiling because (**20**). As a liquid evaporates it tends to cool. This can be shown by (**21**). This illustrates the essential principle of many freezing machines. One is illustrated diagrammatically in (**22**). When some liquid is put in a bottle its vapour

spreads throughout the vessel. The vapour when it has saturated the enclosure exerts a pressure which is decided only by the temperature and is independent of the volume of the enclosure and the presence of other gases or vapours. This pressure is the saturated vapour pressure of the liquid. The vapour pressure increases with rise of temperature until it equals the pressure to which the liquid is subjected. Then the liquid begins to boil. A liquid can therefore boil at any temperature above its melting point if the pressure on its surface is suitably adjusted. The normal boiling point is the temperature for which the vapour pressure equals 76 cm. of mercury (a standard atmosphere). Figure (**23**) shows an apparatus for measuring the B.P. of a liquid.

What factors affect the boiling point of a liquid? (**24**).

Water vapour in the atmosphere is deposited in liquid form when the air temperature falls below the dew point which is therefore defined as (**25**). The relative humidity of the air is defined as (**26**). This can, but rarely is, measured by the chemical hygrometer (**27**) which measures directly (**28**). More usually you use a dew point hygrometer such as (**29**). The relative humidity is calculated from equation (**30**). An even simpler apparatus is the wet and dry bulb hygrometer (**31**), from which the humidity is found by (**32**).

Heat is transmitted by conduction, convection, and radiation. Definitions of these processes are (**33**), (**34**), and (**35**).

The quantity of heat which passes through a slab of a conductor is given by the equation (**36**). Hence the definition of the coefficient of thermal conductivity is (**37**).

Diagram (**38**) shows the convection currents in a domestic hot water system.

An apparatus for detecting small quantities of heat radiation is (**39**).

The quantity of heat given off by convection and radiation from any surface has been found to depend on (**40**). This can be tested experimentally by observing the rates of fall of temperature of known weights of water suspended in calorimeters of known heat capacity.

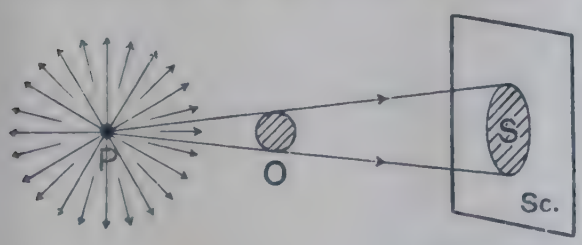
CHAPTER XII

Rectilinear Propagation

Shadows

Everyday observation tells us that light travels in straight lines for we cannot see round the edges of opaque objects and beams of sunlight through small holes seem to have straight edges. Moreover the shadows cast by a very small source of light, such as an electric lamp with a tiny filament, are very sharply defined.

The ways in which shadows are formed by small and large sources of light are illustrated in figs (1), (2) and (3).



P = point source of light.
O = opaque ball.
Sc = screen on which shadow (S) is cast.

FIG. 1

L = source of light smaller than O.
O = opaque ball.
Sc = screen showing complete shadow (*Umbra*) and partial shadow (*Penumbra*).

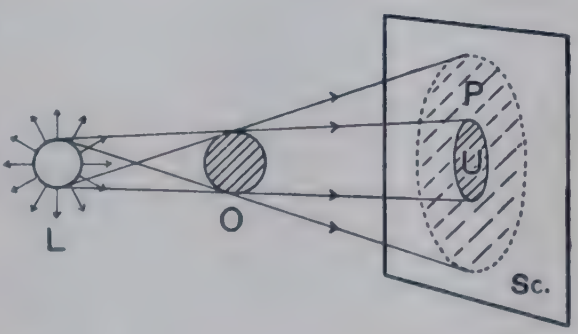
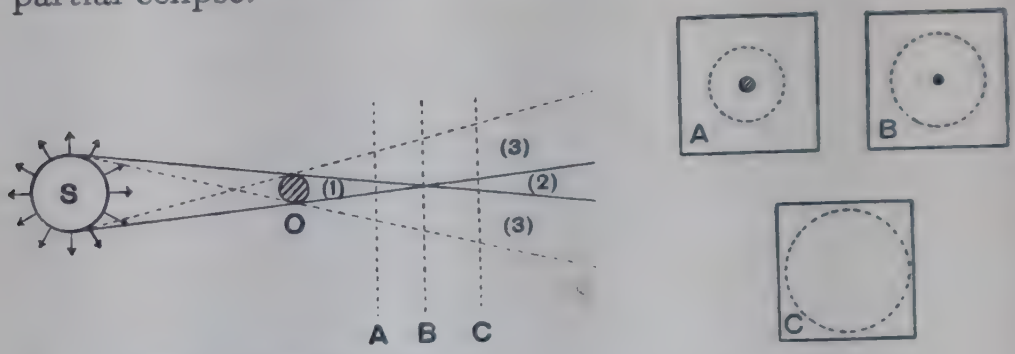


FIG. 2

Fig. 3 illustrates the shadow which the moon produces by cutting off light from the sun. If any part of the earth happens to pass through the cone of complete shadow (1) there is a total eclipse of the sun. Any place which passes

through (2) has an annular eclipse while within (3) there is a partial eclipse.



S = large source of light.
O = opaque ball.
(1) = zone of complete shadow (total eclipse).
(2) = zone of partial shadow (annular eclipse).
(3) = zone of partial shadow (partial eclipse).

FIG. 3

Pinhole Images

The way in which light travels in straight lines is also illustrated by the narrow beams of light which pass through small holes. A small electric flash lamp, which has a tiny filament, is placed about a foot away from a screen of opaque paper in a dark room. A round pin hole in this paper gives a round patch of light on a screen; a triangular hole gives a triangular patch. The size of the patch of light on any screen can evidently be calculated by the simple geometry of similar triangles if we know the size of the hole and the distance of the hole and the screen from the point source. (Fig. 4.)

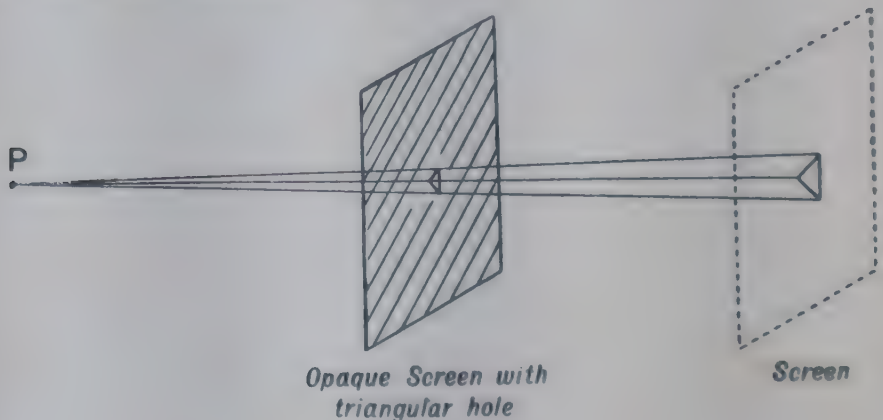


FIG. 4

Two point sources will give two patches of light on the screen, and if the hole is small these will be distinct while if it were big they might overlap.

If the " point source " lamp is replaced by a larger lamp, preferably one with a horseshoe filament, the light on the screen will take the form of the filament inverted—thus forming an " image " of the bright source.

Notice that the size of the image is given by the equation.

$$\frac{\text{Length of image}}{\text{Length of object}} = \frac{\text{Distance of screen from pinhole}}{\text{Distance of source from pinhole}}$$

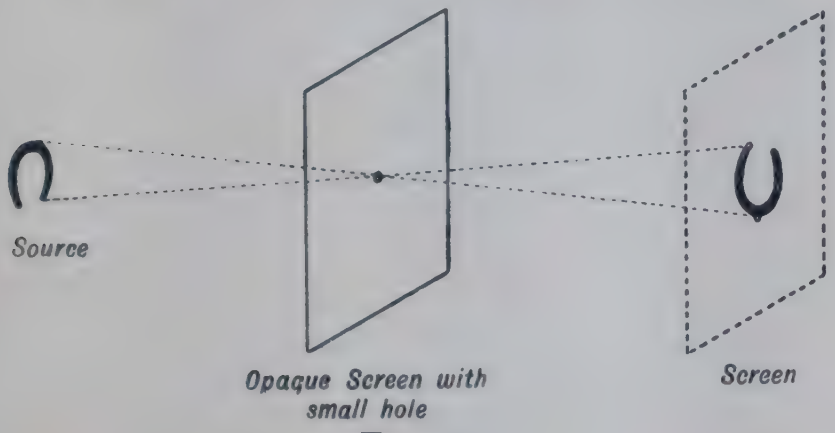


FIG. 5

This is not the same equation as that obtained from fig. 4 which gives the size of the patch of light from every point on the source.

If we replace the opaque screen with its pinhole by a light tight box and use a photographic plate as the screen we have a pinhole camera by means of which photographs can be taken of buildings or objects which do not move during the ten minutes or more required to give the correct exposure.

The picture obtained from a pinhole camera is actually made up of a very large number of small patches each rather bigger than the pinhole. From a short distance such patches are not noticed by the eye although a magnifying glass makes them evident. It is interesting to compare such a picture with an ordinary photographic reproduction in a newspaper, and with a picture taken by a camera fitted with a lens of good quality (see p. 178).

Pinhole images of the sun may often be noticed on the ground under trees. During an eclipse of the sun such images are seen to take the shape of the partially eclipsed source.

Questions on Chapter XII

(1) Describe a laboratory experiment to show that light travels in straight lines.

Explain how a total eclipse of the sun occurs. (N.)

(2) Explain, with diagrams, what is meant by the terms *umbra* and *penumbra* and show how you would demonstrate the effects in the laboratory. What does a person see during an eclipse of the sun if he happens to be in the penumbra of the moon? (D.)

(3) What is a pinhole camera? Explain how images are formed in it, and show how their nature is changed by (*a*) altering the distance of the object, (*b*) increasing the size of the hole. (N.)

(4) If a building 100 yds. away from a pinhole appears on a screen to be 2 in. high and the pinhole is 5 in. from the screen, what is the height of the building?

CHAPTER XIII

PHOTOMETRY

The illuminating power of lamps and the intensity of illumination on surfaces

When we say that two lamps have the same **illuminating power** in a certain direction, we mean that each lamp would, if used alone, produce equal illumination on a surface placed at equal distances from the lamps.

The intensity of illumination on a surface is defined as the amount of light which falls per second on unit area of the surface.

The illuminating power of a source of light was originally expressed in terms of that of a " standard candle," defined as one made of sperm wax burning at the rate of 120 grains per hour. Later other standards were used, and now certain electric lamps are sold as standard lamps of definite candle power. These are expensive and the voltage applied to them has to be exactly regulated if they are to be used for accurate measurements, but a cheap carbon filament lamp, with its approximate candle power marked on it, is good enough for many purposes.

If a 5 c.p. lamp is illuminating a surface placed one foot from the lamp so that the light falls perpendicularly or " normally " on the surface, the intensity of illumination on the surface is called 5 foot-candles. (If the surface is a metre away the illumination is 5 metre-candles.) For ordinary reading it is usual to have from two to five foot candles ; sunlight may produce 10,000 foot candles and the full moon gives about $\frac{1}{50}$ foot candles.

How intensity of illumination varies with distance

Suppose that in an otherwise dark room there is a point source of light of 5 candle power and let there be an opaque screen one inch square placed one foot away from the source (fig. 6).

The shadow of the one inch square cast on another screen d feet from the source will, by geometry, have a side d inches long so that the area of the shadow will be d^2 square inches.

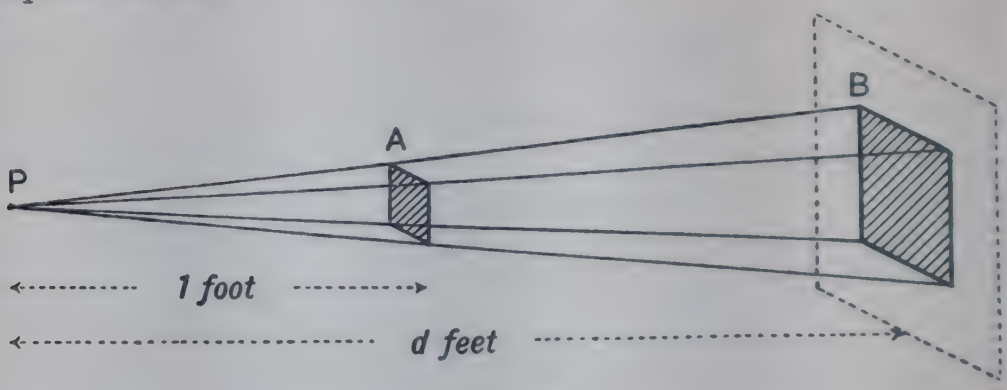


FIG. 6

Now the intensity of the light on the second screen will be given by amount of light falling per second per square inch (see definition of intensity). Also, all the light which falls on the *one square inch* would, if it were allowed to do so, fall on d^2 square inches of the second screen.

Hence

$$\frac{\text{Intensity of illumination at B}}{\text{,, ,, ,, ,, A}} = \frac{1}{d^2} = \left(\frac{\text{Distance PA}}{\text{Distance PB}}\right)^2$$

Hence the illumination on a screen varies ***inversely*** *as the* ***square*** *of its distance from the source of light.*

Most people find it easier to think of the actual intensity of illumination on the screen B when it is written down in foot-candle units. To do this we use the formula :—

$$\textbf{Intensity of Illumination at any point (in ft. candles)} = \frac{\textbf{candle power of source}}{\textbf{square of distance (in feet) from source}}$$

This formula, (and the above proof), is only true if the source is a small one, if the incident light is perpendicular to the surface, and if there are no neighbouring reflecting surfaces which deflect or scatter light. In an ordinary room with white walls the formula gives only very approximate results.

Photometers

An apparatus by which the illuminating power of a lamp can be measured is called a photometer.

Rumford's Photometer 1793

This consists of an opaque rod placed a few inches in front of a white screen. The light from a lamp whose candle power is to be measured is allowed to fall on the screen and a shadow of the rod is noticed. The contrast between this shadow and the screen will obviously vary with the distance of the lamp from the screen. Light from a standard lamp (that is any lamp of known candle power) is now allowed to fall on the screen so that it casts another shadow arranged to be just in contact with the first shadow.

The distances of the lamps from the screen are adjusted until the two shadows seem equally dark. Then the illumination in one shadow due to the unknown lamp is equal to that in the other shadow due to the standard lamp.

Thus Intensity of illumination in the shadows

$$= \frac{\text{C.P. of standard}}{\text{Square of its distance from the screen}}$$

$$= \frac{\text{C.P. of unknown}}{\text{Square of distance of unknown from screen}}$$

This formula is used for all sorts of photometers.

The shadow photometer has the advantage that it can be used in an ordinary room because the general scattered light in the room will illuminate both shadows equally. As in the case of most other photometers its use requires practice and you should always calculate the candle power of an unknown lamp from measurements of at least three pairs of distances to see that your results are reasonably accurate. If the two lamps produce shadows of different colour it is particularly difficult to get consistent results. Many lamps produce different illumination in different directions so it is best to make a small mark on each lamp and to use the same direction throughout a number of experiments.

Bunsen's Grease Spot Photometer (1843)

This uses a screen of white paper with a small spot of grease on it. Two pieces of plane mirror are set at equal angles so that an observer can see both sides of the screen at once (fig. 7) and unless the room is completely darkened

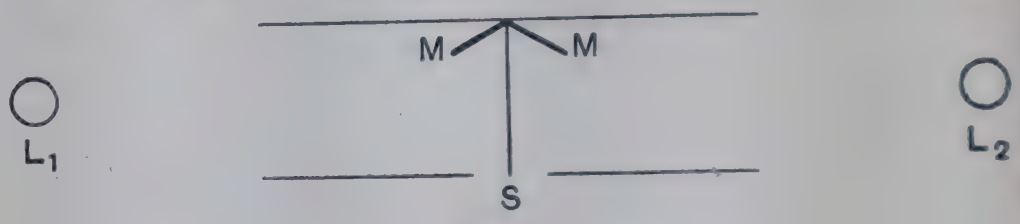


FIG. 7

it is usual to put the screen at the middle of a blackened box with open ends. The box prevents light from other parts of the room falling unequally on the two sides of the photometer screen. The distances of the lamps L_1 and L_2 from the screen S are adjusted until the screen with its small grease spot appears to be equally illuminated on both sides.

When both sides of the screen are equally illuminated we use the usual photometer formula

Then

$$\text{Intensity of illumination} = \frac{\text{C.P. of } L_1}{\text{Square of distance } L_1S} = \frac{\text{C.P. of } L_2}{\text{Square of distance } L_2S}$$

The only use of the grease spot is as an aid to the observer in making his judgment of equal illumination.

The remarks made about the difficulty of using the shadow photometer apply also to the grease spot photometer.

Illumination meters or foot-candle meters

These are used to measure the intensity of illumination in foot candles at some place in a room. The simplest form of foot-candle meter consists of a semi-transparent screen with a number of small grease spots fixed on top of a box in which an electric lamp is placed so that the illumination it produces at the various spots varies say from 10 to 1

foot candles. If the instrument is placed on the bench it is possible to judge which of the spots appears least visible and the corresponding illumination is read off on the scale of the instrument.

The most modern illumination meters, now extensively used as exposure meters by photographers, employ an electrical circuit which produces different currents when the active surface of the instrument is exposed to light of different intensities. Such photo-electric meters can be calibrated to read directly in foot candles and, as no judgment of equal intensity has to be made, they require no skill in use and give very consistent results.

Questions on Chapter XIII

(**1**) Explain the terms—*illuminating power, intensity of illumination.*

Define units in which these quantities may be measured.

At what distance from a screen must a lamp of 50 candle-power be placed to give the same illumination as a 30 candle-power lamp at 3 ft. distance? (W.)

(**2**) Explain the meaning of (*a*) illuminating power, (*b*) intensity of illumination, and state the unit in which each is measured.

Describe the construction and explain the use of some form of photometer.

Two lamps, *A* and *B*, are balanced on a photometer when *A* is 60 cm. and *B* is 40 cm. from the photometer head. *B* is a 40 candle-power lamp; find the candle-power of *A*.

A reflector is then placed close to *B* so that it throws more light from *B* on to the photometer head, and in order to obtain a balance *A* has to be placed at 50 cm. from the photometer head. Calculate the apparent increase in candle-power of *B* due to the reflector. (O.)

(**3**) How would you show that light is propagated according to the "inverse square law"?

Find the candle-power of an incandescent bulb which at a distance of 6 ft. gives the same illumination as a 32 candle-power bulb at 4 ft. 6 in. (O. & C.)

(**4**) Describe a photometer and explain the method of comparing the illuminating powers of two sources of light.

Two lamps *A* and *B* have illuminating powers in the ratio 4 : 5. When *A* is 2 ft. from the photometer screen where must *B* be placed to give equal illumination? (L.)

(**5**) Explain how Rumford's shadow photometer can be used to compare the illuminating powers of two small electric lamps. Give the principle of the method.

Two lamps of 24 and 6 candle-power respectively are placed 60 cm. apart, and a cardboard screen is placed perpendicular to the line joining the two lamps. Find the position (*a*) between the lamps, (*b*) beyond one lamp, where the screen is equally illuminated by both. (N.)

(**6**) A room is lit by two electric lamps of equal candle-power which are hung 3 ft. above a table and 4 ft. apart. Compare the illumination at a point directly below one lamp when both are lit, with that given by each singly. (B.)

(**7**) Explain what is meant by the description "100 candle-power" as applied to a lamp.

Describe in detail any **one** practical method for comparing the candle-powers of two lamps. Explain whether your measurements should be carried out in a darkened room or whether this is not necessary. (N.)

(**8**) Two lamps, of candle-power 1 and 16 respectively, are set up 3 feet apart. Find where a screen must be placed between them and on the line joining them to be equally illuminated by the two lamps. (O.)

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CHAPTER XIV

THE NATURE AND EFFECTS OF LIGHT

The Nature of Light

It is found convenient to regard light as a disturbance consisting of very tiny waves of various wavelengths and frequencies.

The *frequency* or number of waves which pass any point in one second is thought to decide the colour of the light as seen by the human eye, blue light having about double the frequency of red light.

The *wave length* depends on the transparent medium (e.g., air, glass, water) in which the light is travelling; in air, the wavelength of blue light is about $\frac{1}{40000}$ inch, while that of red light is about $\frac{1}{30000}$ inch. Light waves are thought to be electromagnetic in nature because very similar waves, used in wireless telephony, are produced by electrical apparatus.

Light waves can travel through the almost empty space between the sun and stars and the earth, through the best vacuum we can make inside an electric lamp bulb, or through innumerable " transparent " substances. Through empty space, measurements have shown that the speed of the light waves of all frequencies is about 186,000 miles per second. The waves of visible light are so small that any train of them appears to travel in a straight line and so the path of a group of the waves is bounded by straight lines. So we speak of beams of light and of the very smallest beam as a *ray of light*. For many years, the fact that light travelled in straight lines was explained by supposing that light consisted of tiny particles ; then, about 100 years ago, this theory fell into disuse because the wave theory was more useful in explaining the behaviour of light. During the past thirty years it has been found that light sometimes appears to behave as a wave disturbance and sometimes it shows some of the properties of particles or perhaps of separate trains of waves.

Effects produced by light

When light passes through empty space it is " invisible " but when it falls on material substances the light scattered from their surfaces makes them visible to our eyes. Even in an apparently empty room the path of a ray of sunlight is usually clearly visible because of the light scattered to the eye from the innumerable specks of dust floating in the atmosphere.

When light falls on a rough surface it is usually scattered in all directions so that a large surface of rough paper appears more or less equally visible in all directions. If the surface appears to be " coloured " we suppose that of the light waves which fall on it, it scatters mainly those of its characteristic colour and absorbs partially or completely those of other frequencies. We shall consider " colour " more in detail later. A black surface can be thought of as one which absorbs all the light waves which are falling on it.

The very smooth surfaces of mirrors show a different effect. They cause beams of light to be reflected in definite directions.

Smooth transparent surfaces allow much of the light incident on them to pass into the transparent medium, generally with a change of direction which we shall see later is due to a change in speed of the waves.

Most surfaces show a rise of temperature when light falls on them. This is particularly noticeable when we touch a blackened metal surface which has been standing in bright sunlight. We say that the light which has been absorbed by the surface has been converted into heat. Heat is supposed to be due to the vibrations of the tiny particles of which all substances are believed to be composed. We imagine that the tiny light waves, as they fall on the surface particles, set them in vibration and so, when touched the surface feels hotter than it would have been if no light had fallen on it. This effect is also easily shown by exposing a thermometer, preferably one with a blackened bulb, to sunlight.

We have already said that some surfaces produce electrical effects when light falls on them. This effect is the basis of the action of the photo-electric cells used in measuring illumination and in those used in the production of sound from films.

CHAPTER XV

THE REFLECTION OF LIGHT

Laws of Reflection

Experiments show that, in all cases of reflection, the behaviour of a reflected ray of light can be summarised in terms of the two " laws of reflection."

(1) The reflected ray lies in the plane which contains the incident ray and the straight line drawn perpendicular to the reflecting surface at the point of incidence. (This straight line is called the " normal " to the surface at this point.)

(2) The angle of reflection (between the reflected ray and the normal) equals the angle of incidence (between the incident ray and the normal).

Sometimes you are asked how you would " prove " (or better, illustrate) these two laws. You should yourself have made experiments to illustrate them—the most convincing experiments can be done with an actual beam of light but there are also experiments with pins. Be sure that you can describe in detail some suitable experiments.

The Image formed by a Plane Mirror.

Suppose there is a luminous point source S in front of a plane mirror. Then any ray of light SM will, after reflection at M, become the reflected ray MP.

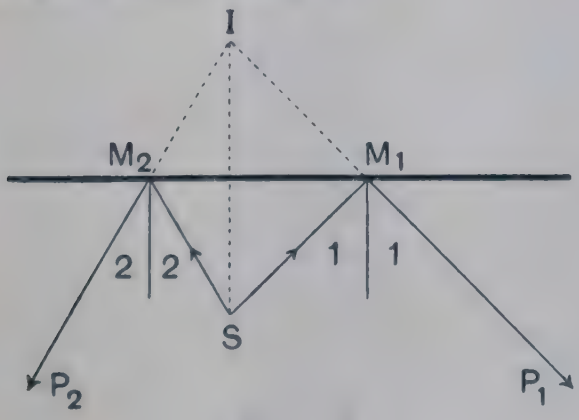

FIG. 8

Fig. 8 shows two such rays and since the various sets of angles marked are equal angles it is evident that the triangle IM_2M_1 is equal in all respects to SM_2M_1. (Be sure that you can give a clear detailed proof of this on paper.) Hence all the rays from this point source seem, after reflection, to be coming from the point I. This point is called the "***image***" of the point source S. With plane mirrors this image is called a ***virtual image*** because the reflected rays only ***appear to come*** from I but have never actually ***passed through it.*** When we deal with curved mirrors we shall find that they can produce either ***virtual images*** of this sort or "***real***" ***images*** through which the reflected light has actually passed.

Notice that the position of this virtual image in a plane mirror is most easily described by saying that it is as far behind the mirror surface as its object is in front of that surface, and along the perpendicular from the object to the surface.

The image of an object placed in front of a plane mirror

We have considered only the image of a point source. When an actual object is put in front of the mirror every point on it is regarded as sending out or scattering rays of light. Thus the image of the object is formed of the point images of all its parts.

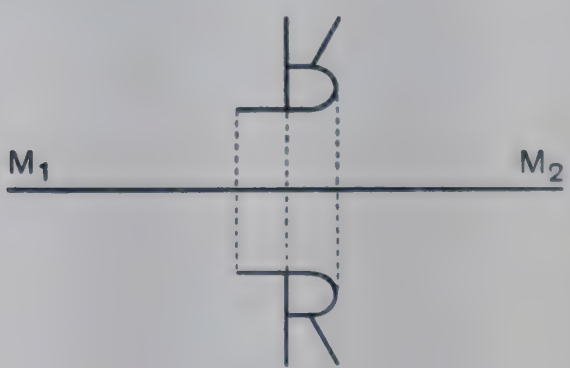

FIG. 9

Fig. 9 shows, in plan, a letter R lying in front of a perpendicular piece of plane mirror M_1M_2.

Since the image of each point on R is as far behind the

mirror as each point source is in front of it, we see that the complete image is exactly the same size as the object but that it is " laterally inverted."

This is characteristic of the images seen in a plane mirror. If you look at your own image in such a mirror and move your right hand you will notice that the image moves its left hand !

The position of an eye which can see the image

If an eye is able to see an image formed by a plane mirror it must be receiving rays of light reflected from some part of the mirror's surface. Hence to decide in what position the eye should be placed, first mark the position of the image, since that depends only on the positions of the object and of the mirror.

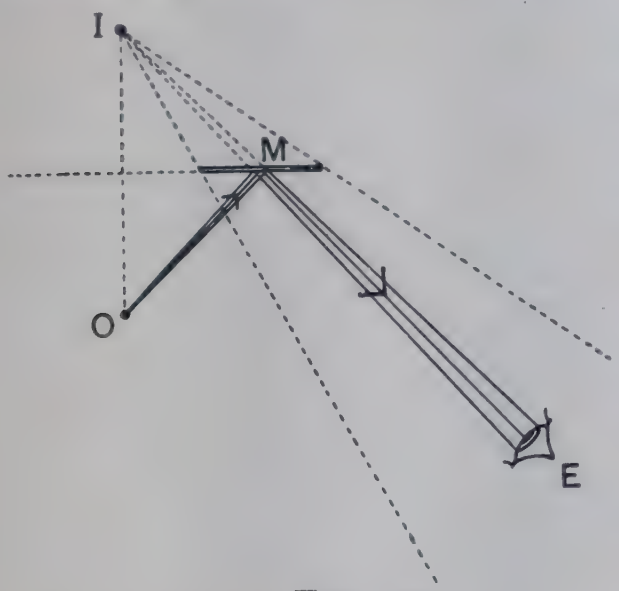

FIG. 10

Then straight lines drawn from the image through the boundary of the mirror will enclose the space within which the eye must be placed (fig. 10).

The pencil of rays by which an eye sees an image

The phrase " pencil of rays " is used to describe a narrow beam of light converging to or diverging from an object or an image. Suppose that in fig. 10 the eye E is in a position from which it can see the image I. Then it must be receiving the pencil of rays ME apparently diverging from I. Hence the actual pencil of rays must be OME. This construction must be clearly understood since it illustrates the general method used later in diagrams of optical instruments of many kinds. First we find the exact position of the image, then join it to the eye and finally draw the original pencil of rays.

Reflections in inclined plane mirrors

Mark on a sheet of paper a circle and shade in a small segment as shown.

Now place the two mirrors perpendicular to the paper so that they are in contact at Q and enclose a known angle M_1QM_2.

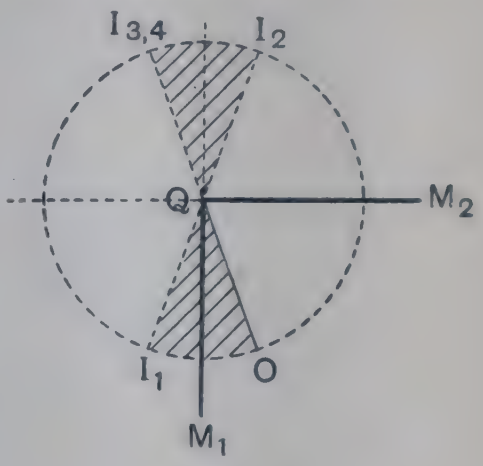

FIG. 11

If this angle is 90° fig. 11 is obtained while an angle of 60° gives fig. 12.

In fig. 11 you can think of the final image $I_{3,4}$ either as the image of I_1 or of I_2 and if you move the mirror slightly so that the angle becomes just less than 90° you will see that the two images are then distinct from each other. Similarly with $I_{5,6}$ in fig. 12.

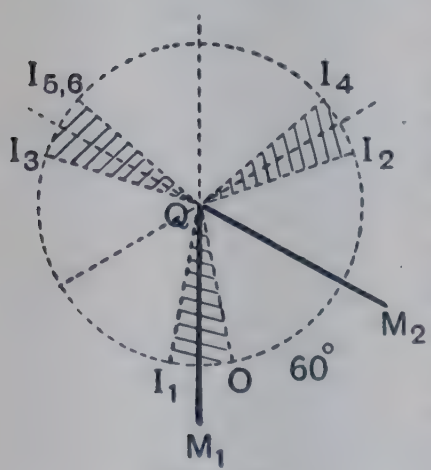

FIG. 12

You should be able to draw a diagram to illustrate how an eye will see any of these images. For I_1 or I_2 the diagram will be similar to fig. 10. For $I_{3,4}$ there will be two reflections of the pencil of rays which comes to the eye from the object O. The construction is shown in fig. 13 and is a simple extension of that already described.

The image seen by an observer looking at himself in two mirrors inclined at right angles

There is one interesting fact about the image $I_{3,4}$ when the observer is looking at his own image. If with two suitably placed large mirrors you watch the effect of moving your left arm you will see that this particular image moves its left arm. Compare this effect with that observed when

you are looking at either I_1 or I_2. The reason is indicated by the positions of the small crosses in fig. 13.

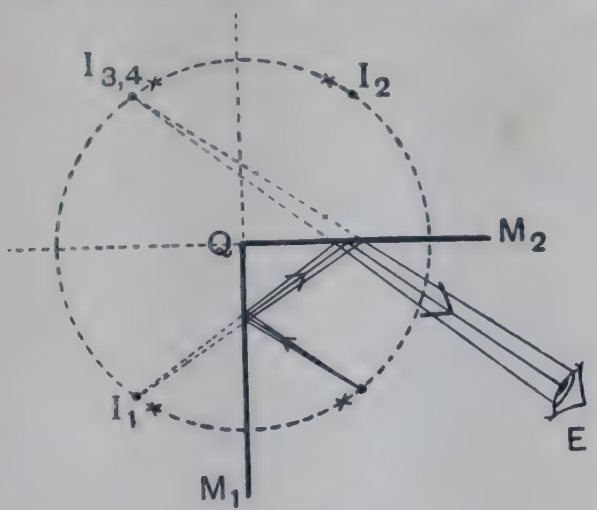

FIG. 13

The Kaleidoscope

Three mirrors inclined at 60° to one another, so as to form a triangular prism, give a still more complicated pattern of hexagonal symmetry. A kaleidoscope is made by so arranging three strips of mirror. A few small bright coloured objects placed between the mirrors give geometrical patterns sometimes of considerable interest and beauty.

Images seen in parallel plane mirrors

The number of images formed in inclined plane mirrors increases rapidly as the angle between the mirrors is diminished.

With plane mirrors which are exactly parallel there are theoretically an infinite number of images but since every reflection is accompanied by some loss of light, only a limited number of these images are visible. The positions of these images are illustrated by fig. 14 which shows also a pencil of rays, by which an eye sees the third image in one of the mirrors.

The images seen in a thick glass mirror by an observer who looks into the mirror from the side are produced by successive reflections but here the effect is complicated by the change of direction (refraction) which occurs when

light passes from one transparent substance to another. The diagram (fig. 15) illustrates the effect but it cannot be clearly understood until a knowledge of refraction has been acquired.

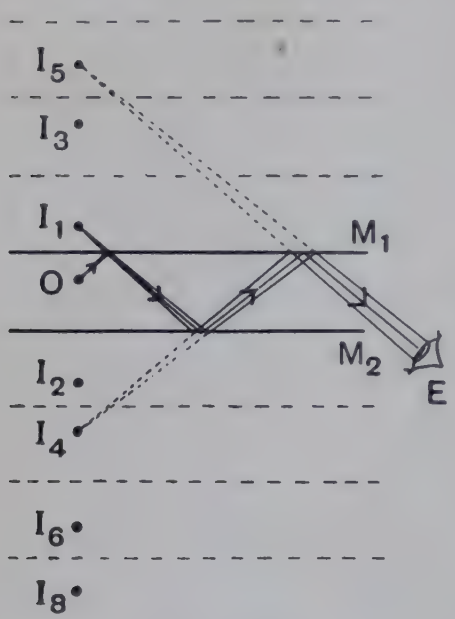

FIG. 14

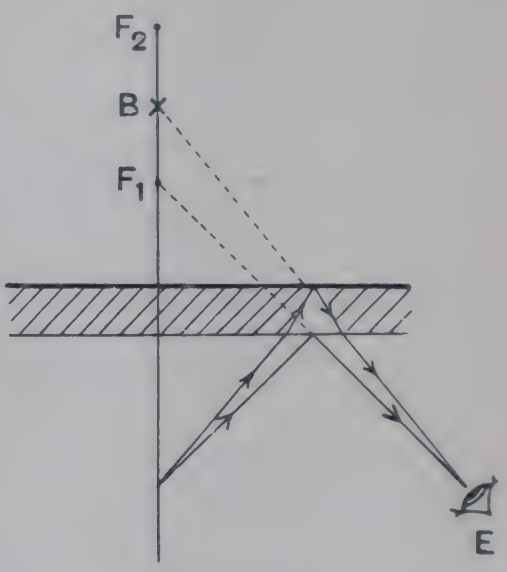

FIG. 15

The effect of rotating a plane mirror

In fig. 16 an incident ray PQ falls on a plane mirror QM_1 at an angle of incidence A. The reflected ray QR_1 will also make $\angle A$ with the normal QN_1. Hence $\angle PQR_1$ is 2A.

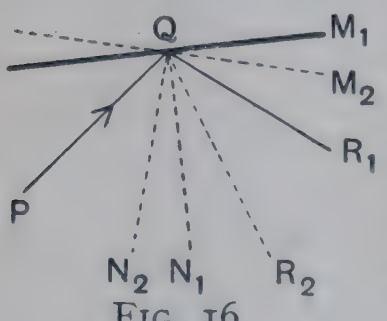

FIG. 16

Now let the mirror be moved through an angle B to the position QM_2. The normal now is QN_2 so that the new angle of incidence PQN_2 is (A − B). Hence N_2QR_2 is also (A − B) and PQR_2 is 2(A − B).

Thus the angle R_1QR_2 turned through by the reflected ray is $PQR_1 - PQR_2 = 2A - 2(A - B) = 2B$

Hence when a mirror is rotated a beam of light incident on it in a fixed direction has its reflected beam turned through twice the angle of rotation of the mirror.

This effect takes place in the Sextant, an instrument used by navigators for measuring the angle between the sun (or the moon or a star) and the horizon. It also occurs with reflecting galvanometers used for electrical measurements.

The angle through which a beam of light is deviated when it is reflected successively at two inclined mirrors

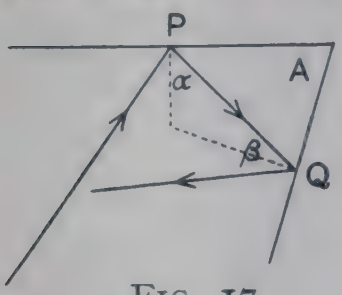

FIG. 17

The change of direction at the first reflection at P is $180 - 2\alpha$

At Q the change is $180 - 2\beta$

Hence total deviation is

$$360 - 2(\alpha + \beta) = 360 - 2A$$

Hence the deviation is 360—twice the angle between the two plane mirrors.

Questions on Chapter XV

(**1**) State the laws of reflection of light and deduce from them the relation between the image distance and the object distance in the case of a plane mirror.

Describe how you would verify your result experimentally. (L.)

(**2**) A point source of light is placed in front of a plane mirror. Trace the pencil of rays by which an observer sees the image of the point source.

A boy 4 ft. 3 in. in height is standing at a distance of 3 ft. from a vertical mirror 2 ft. in length, the upper edge of which is 5 ft. from the floor. Taking the boy's eye level to be 4 ft. above the floor, determine, with the help of a diagram, how much of his height will be seen by him in the mirror and what length of the mirror is useful to him. (N.)

(**3**) State the laws of reflection of light, and describe how they may be verified experimentally.

What are the characteristics of the image formed by a plane mirror?

An electric strip-light 2 ft. in length is 2 ft. vertically

above, and parallel to, a horizontal strip of plane mirror 1 ft. long. Their centres lie in the same vertical line. Find the length of (*a*) the image of the strip-light in the mirror, and (*b*) the reflected patch of light on the ceiling 2 ft. above the strip-light. (B.)

(**4**) State the laws of reflection of light and explain how you would test them experimentally.

Two plane mirrors A and B are arranged at right angles to each other. A small object is placed between the mirrors, but nearer to A than to B. Give a diagram showing the positions of any images which can be seen. Draw the paths of the pencils of light by which an image is seen in A by reflection (*a*) at A only, (*b*) at both mirrors. (D.)

(**5**) An object is viewed in a plane mirror by means of a telescope. On rotating the mirror about an axis perpendicular to the axis of the telescope a second object is brought into view. Determine the relation between the angle of rotation and the angle subtended at the mirror by the two objects. (L.)

(**6**) State the laws of reflection of light.

A ray of light is reflected from two plane mirrors in succession. Find the angle between the mirrors for the ultimate path of the light to be at right angles to its original path. (L.)

(**7**) State the *laws of reflection* of light, and prove that the image formed by reflection in a plane mirror is as far behind the mirror as the object is in front.

Two plane mirrors are inclined at an angle to each other. A ray of light parallel to one of the mirrors travels after two reflections parallel to the other. Find the angle between the mirrors. (O. & C.)

(**8**) Explain the following observation, and illustrate your answer by means of a diagram :

When a lighted candle is held in front of a thick plate glass mirror (silvered at the back), several images of the candle can be seen one behind the other, of which the second is the brightest. (C.)

(**9**) How many images are formed of an object placed

between two plane mirrors inclined to each other at an angle of 60° ?

Trace the path of a pencil of light from a point $\frac{1}{2}$ in. from one mirror and 1 in. from the other to an eye 2 in. from each of the mirrors when the light suffers (*a*) 2 reflections, (*b*) 3 reflections.

CHAPTER XVI

REFLECTION AT SPHERICAL MIRRORS

When a ray of light falls on a concave or a convex spherical surface it can be shown by experiment that the two "laws of reflection" (see p. 135) tell us exactly what happens to the reflected ray.

The "***radius of curvature***" of such a mirror is defined as the distance from the mirror P to the centre C of the sphere of which the mirror's surface forms a small part (fig. 18). The simple theory which we shall use assumes that the part of the spherical surface in use is very small compared to the length of this radius of curvature. A straight line drawn from the middle of the mirror's surface through its centre of curvature is called the ***principal axis.*** To avoid confusion between the centre of curvature and the middle point on the mirror's surface the latter is often called its ***pole*** (P).

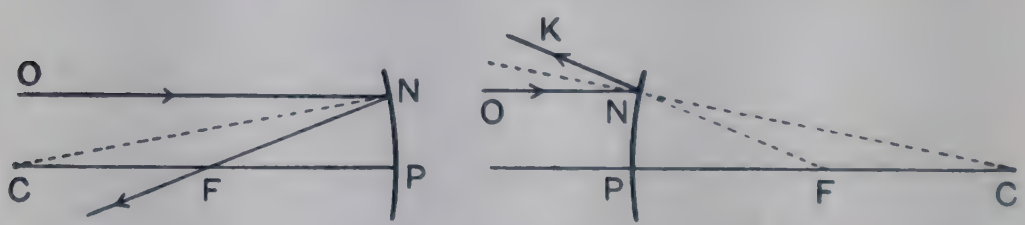

Fig. 18

The distance across the piece of mirror surface which is in use is often called its ***aperture.***

If a beam of sunlight is allowed to fall on a concave mirror of small aperture in a direction parallel to its principal axis, the reflected rays of sunlight all pass through a point F which is half-way between P and C. An object placed at F gets very hot when the reflected light and heat of the sun are concentrated on it. Hence the point F is called the ***principal focus*** of the mirror. The length FP is called the ***focal length*** of the mirror.

With a convex mirror the principal focus is behind the

mirror. Since the reflected light does not pass through it but only appears to come from it, the focus of a convex mirror is said to be a virtual focus whereas that of the concave mirror is a real focus.

Proof that the principal focus lies half-way between the mirror surface and its centre of curvature

The principal focus is such that any incident ray ON parallel to the principal axis CP is reflected along NF (or with a convex mirror appears to come from FN) (fig. 18). NC is normal to the mirror at N.

Hence $\angle ONC = \angle FNC$ (law of reflection)

But $\angle ONC = \angle FCN$ because ON is parallel to CF

$\therefore \angle FNC = \angle FCN$ So that FN = FC

Now if we consider only mirrors of aperture small compared to their radii of curvature, FN is nearly equal to FP.

Hence PF is nearly equal to FC.

The position of the image of an object

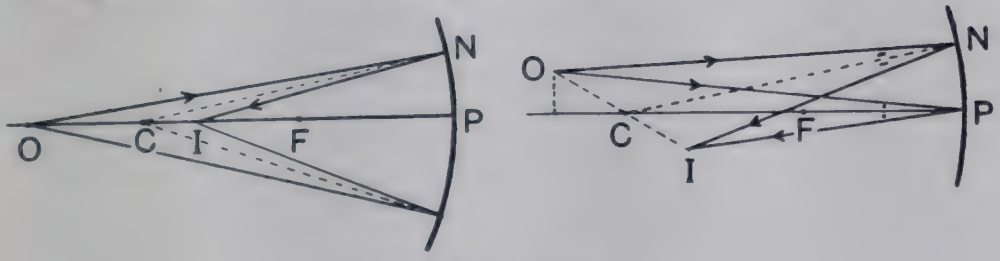

FIG. 19

If rays of light come in all directions from a point-source O to the mirror it is found that, with a mirror of small aperture, all the reflected rays are concentrated again at a point I (or appear to come from I when the image is a virtual one).

If O is on the principal axis, I will also be on it. If O is not on this axis, then I will still lie on the line OC produced, since OC will be the direction of a ray of light which must fall on the mirror perpendicular to its surface and so must be reflected back along its own path.

To avoid drawing a series of angles CNI equal to a series of angles ONC, in order to find the point I where all the reflected rays meet it is sufficient to draw two of the three rays shown in fig. 20.

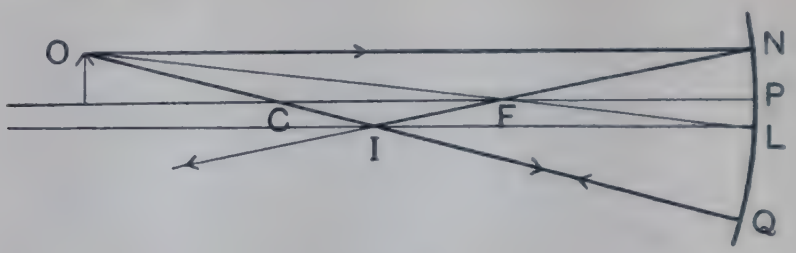

FIG. 20

The ray from O through C is reflected back along its own path.

The ray ON parallel to the principal axis passes after reflection through F.

The ray OF through the focus is reflected along LI parallel to the principal axis.

This diagram will give the exact position of the image I for either concave or convex mirrors provided the point O is not on the principal axis and provided the aperture of the mirror is small compared to its radius. Since we cannot draw very accurate scale diagrams with the object and the aperture shown very small, we generally use different scales in the two directions PC and PN just as is often done when drawing graphs. When this is done, the mirror surface looks very flat and it is essential in these optical scale diagrams that NP should be exactly perpendicular to PC. We shall know whether the diagram is intended to represent a concave or a convex mirror from the positions of the centre of curvature (C) and the principal focus (F).

Scale diagrams need a good deal of practice and every student should have tried to draw such diagrams for various positions of the object. Different distances of the object from a given mirror give rather different figures and experience shows that if the distances of the object are thought of in terms of the focal length of the mirror, it is easier to remember what sort of image is obtained. The student should use a ruler and draw diagrams for an object placed at distances 3f, 2f, $\frac{3}{2}$f and $\frac{1}{2}$f.

The pencil of rays by which an eye sees the image

An eye can see the image if the eye is in the space between lines from the image to the edges of the mirror.

The pencil of rays by which the eye sees an image can be drawn by joining the edges of the pupil of the eye to the image and producing these lines to cut the mirror. This

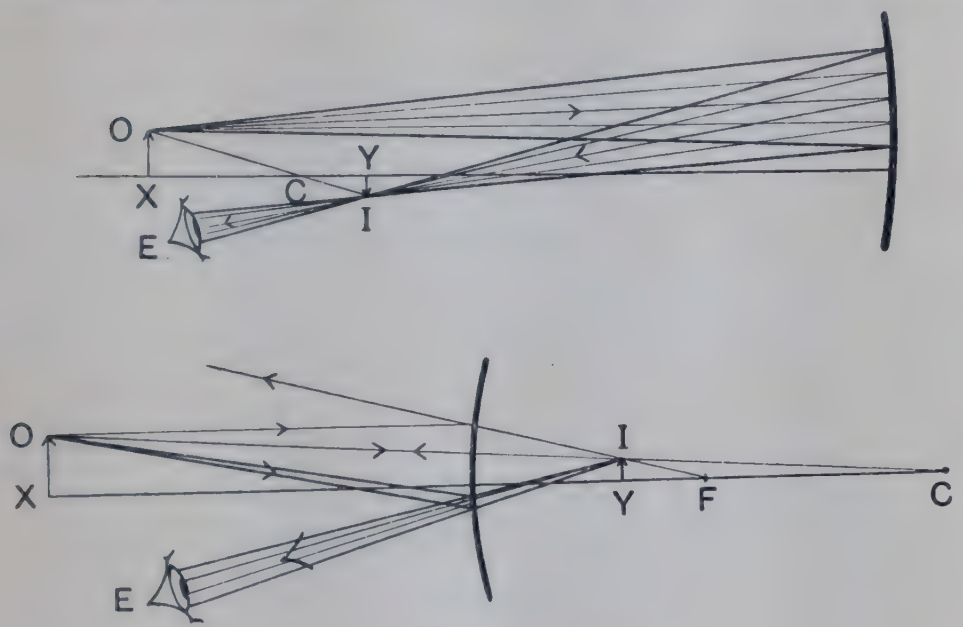

FIG. 21

gives the reflected rays. The incident pencil is obtained by joining the object point to the parts of the mirror surface from which the reflected rays come. (Fig. 21.)

Since with a convex mirror the images of all objects are virtual, erect, and lie between the mirror surface and its virtual focus a convex mirror is generally used in motor-cars to show the road behind the car. Because the virtual images are formed near to the surface of a convex mirror the space in which the eye can be placed is much wider than the corresponding space with a plane mirror.

Conjugate Foci

If the object is at the point I it will form an image at O since the path of a ray of light is always reversible. Thus any two image and object points I and O are said to be "*conjugate foci.*"

The height of an image

The relation between the lengths of an extended object and its image is evident if we remember that a ray from O to the pole of the mirror must be reflected through I. Thus in fig. 22 OPX and IPY are equal angles so that

$$\frac{IY}{PY} = \frac{OX}{PX}$$

$$\textbf{Hence} \quad \frac{\textbf{Height of image}}{\textbf{Height of object}} = \frac{\textbf{Distance of image from mirror}}{\textbf{Distance of object from mirror}}$$

This formula gives the diameter of the image of the sun formed at the focus of a concave mirror if the distance and diameter of the sun and the radius of curvature of the mirror are known.

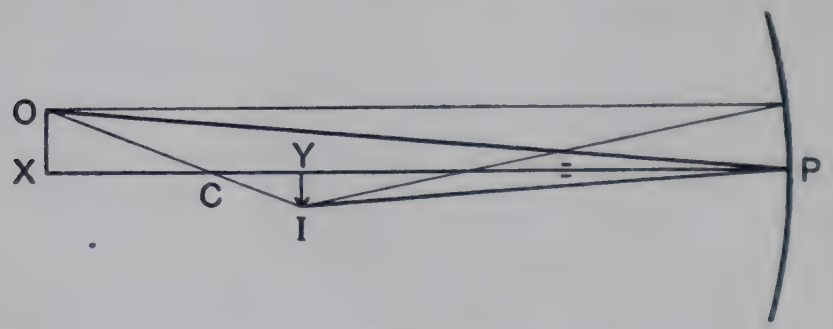

FIG. 22

Calculations of the distances of images from mirrors

Accurate scale diagrams take time to draw and it is quicker to make a rough diagram to illustrate what happens and then to calculate the position and size of an image from equations derived from the geometry of the diagram. To do this use is made of the two equations :

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\textbf{and} \quad \frac{\textbf{height of image}}{\textbf{height of object}} = \frac{v}{u}$$

Here v represents the distance of the image from the mirror surface.

u represents the distance of the object from the mirror surface.

f represents the focal length of the mirror.

The focal length (half the radius of curvature) is usually thought of rather than the radius because it is so easy to find it experimentally with a concave mirror and because a lens has also a focal length but has not a radius of curvature.

Geometrical proof of the mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

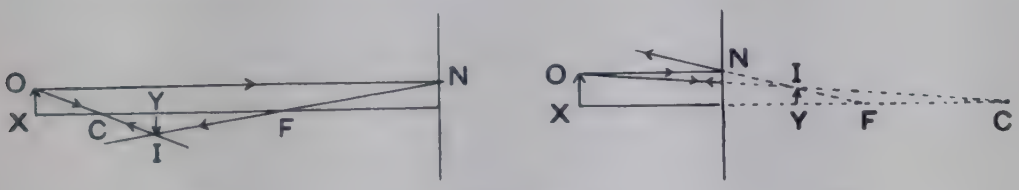

FIG. 23

In fig. 23 the usual rays ON, NI and OCI are drawn.

Here PX = u PY = v PF = FC = f

Because OCX and ICY are similar triangles $\frac{IY}{OX}=\frac{CY}{CX}=\frac{2f-v}{u-2f}$

Because NFP and IFY are similar triangles

$$\frac{IY}{OX}=\frac{IY}{NP}=\frac{FY}{FP}=\frac{v-f}{f}$$

Hence $\frac{2f-v}{u-2f}=\frac{v-f}{f}$

$$2f^2 - fv = vu - 2fv - fu + 2f^2$$

$$fu + fv = vu$$

Divide through by fuv to get

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Use of positive and negative signs

This proof has been given for fig. 23 (*a*). To deal with fig. 23 (*b*) we have to adopt some convention about the sign of distances measured from the mirror. A simple convention for elementary work is to call distances measured on the same side as the object, positive and on the other side, negative. Then the focal length of a concave mirror becomes positive and that of a convex mirror negative.

Examples

(1) An object 2 cm. high is placed 30 cm. from a concave mirror of focal length 10 cm. Calculate the position and size of the image.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\therefore \frac{1}{v}+\frac{1}{30}=\frac{1}{10} \quad \therefore \frac{1}{v}=\frac{1}{10}-\frac{1}{30}=\frac{2}{30}$$

Hence $v = 15$ cm.

This result being positive the image is on the same side of the mirror as the object. Thus it is real and inverted.

$$\text{Further } \frac{\text{Height of image}}{\text{Height of object}} = \frac{v}{u} = \frac{15}{30}.$$

$\therefore$ The image is 1 cm. high.

(2) An object is 10 in. from a convex mirror of radius of curvature 8 in. Find the position of the image.

In this case the centre of curvature is behind the mirror and therefore the focal length is $-(\frac{1}{2} \times 8) = -4$ in.

$$\therefore \frac{1}{v}+\frac{1}{10}=-\frac{1}{4}$$

$$\therefore \frac{1}{v}=-\frac{1}{10}-\frac{1}{4}=-\frac{14}{40}$$

$$\therefore v = -\frac{20}{7} = -2\tfrac{6}{7} \text{ in.}$$

Thus the image is behind the mirror, i.e., it is a virtual image.

N.B. In working out examples on mirrors and lenses it is advisable that the student should not prefix a sign to the unknown quantity, since the equation will give both the magnitude and the *sign* of the unknown.

Another convention concerning the use of positive and negative signs is given on page 173.

Experimental work with concave mirrors

Measurement of focal length

(1) This can be found quickly, to within a quarter of an inch, by catching on a strip of paper the inverted real image of a distant object.

(2) A bright object placed at a distance greater than the focal length will give a real image which can be caught on a screen of white paper. This enables us to measure accurately, to $\frac{1}{10}$ inch, corresponding values of u and v. Then, by substituting in the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we can calculate a number of values of f and take the average.

The magnification, defined as $\frac{\text{height of image}}{\text{height of object}}$, can also be measured directly in this experiment. It should, in each case, be equal to the value of $\frac{v}{u}$.

(3) For accurate measurements a pin is often used as the object and another pin is placed so that it coincides with the image. When this has been done, it will be noticed that however much you move your head, you find that the second pin seems to keep in contact with the image of the first one.

You can be sure of getting results quickly if you place your object first at distances about equal to 3f, 2f and $\frac{3}{2}$f, obtaining f approximately by method (1).

It is convenient to remember that with a concave mirror the image is real, inverted and diminished if the object lies beyond twice the focal length ; real, inverted and magnified when the object lies between the centre of curvature and the focus ; virtual, erect and magnified when the object is between the focus and the mirror surface.

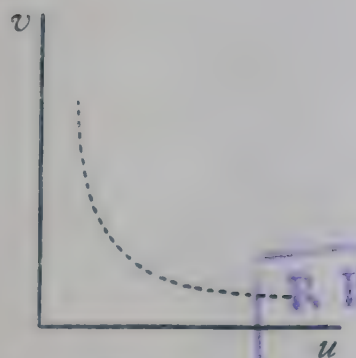

FIG. 24

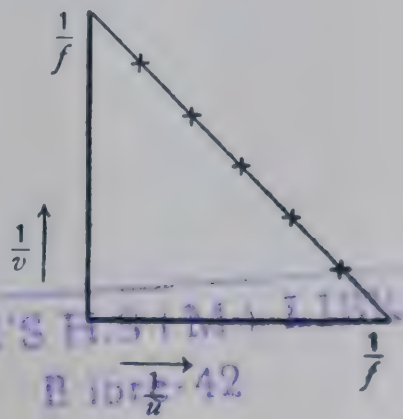

FIG. 25

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Graphs of values of v and u appear for real images as shown (fig. 24). (It is well to choose the same scale for v as for u.)

The value of f can be read off from the graph since when $u = v$ both equal 2f, because then $\frac{1}{u} + \frac{1}{u} = \frac{1}{f}$

Graphs of $\frac{1}{v}$ plotted against $\frac{1}{u}$ are straight lines which cut the axis at distances equal to $\frac{1}{f}$. (Fig. 25.)

Experimental work with convex mirrors is more difficult since the images are virtual being behind the mirror surface between the mirror and its virtual focus. (A method of finding the focal length of a convex mirror is given on page 173.)

Questions on Chapter XVI

(1) A small flashlamp bulb is moved along the axis of a concave mirror and the beam of reflected light falls on a screen held at right angles to the axis. Describe how the size of the circular disc of light on the screen will vary as the bulb is moved and explain the changes by means of diagrams. You may take the mirror of about 1 metre focal length and the screen as placed about 3 metres from the mirror.

When, in such an experiment, the diameter of the light patch on the screen is the same as that of the mirror, what relation holds between the distance of the lamp from the mirror and the radius of curvature of the mirror? (D.)

(2) What do you understand by the *conjugate foci* of a concave mirror?

Describe a method of finding accurately the focal length of a concave mirror.

Explain carefully, with the aid of a ray diagram, how a concave mirror can be used to produce a magnified erect image.

An object placed 28 cm. in front of a *convex* mirror produced an image 12 cm. behind the mirror. What is the

radius of curvature of the mirror, and how does the size of the image compare with that of the object? (O.)

(**3**) Distinguish between a real and a virtual image.

At what distance from an object should a concave mirror of 6 in. radius of curvature be placed to give an image magnified three-fold when the image is (*a*) real, (*b*) virtual? Illustrate your answer by diagrams drawn to scale. (L.)

(**4**) An object 4 cm. long is placed with its centre on the axis of a concave spherical mirror, of 40 cm. radius of curvature, with its length at right angles to the axis. Find, by graphical construction, the position and size of the image formed when the distance of the object from the mirror is (*a*) 15 cm., (*b*) 55 cm., and state in each case the nature of the image. Explain and justify the methods you employ. (C.)

(**5**) Explain, with the aid of a diagram, how a real image differs from a virtual image.

An image four times as high as the object is formed on a screen by a concave mirror. If the distance of the object from the mirror is 60 cm., what is the distance of the image from the mirror? Find also the focal length of the mirror. (C.)

(**6**) Show that the focal length of a concave mirror is equal to half its radius of curvature. How would you find the focal length of such a mirror experimentally?

The reflecting mirrors of motor cars are made convex. Why is this? Draw a diagram for such a mirror, showing the positions of the eye, the object and the image. (O. & C.)

(**7**) An object 1 cm. long is placed 5 cm. from a convex mirror having a radius of curvature of 20 cm. Find the position and size of the image.

Will it be erect or inverted? (O.)

(**8**) A stick, which is 4 inches in length, is held horizontally so that its nearer end is 2 feet in front of a convex mirror whose radius of curvature is 8 inches. Where will the image of the stick appear to be and what will be its apparent length? (D.)

CHAPTER XVII

THE REFRACTION OF LIGHT

WHEN a narrow beam of light falls on a plane surface of some transparent substance some of the light is reflected and goes back at an angle of reflection equal to the angle of incidence. Another part enters the transparent substance and travels on as a ***"refracted beam"*** usually in a direction different from that of the incident beam. (Fig. 26.)

Laws of Refraction—Snell 1621

(1) The refracted ray lies in the plane which contains the incident ray and the straight line drawn perpendicular to the refracting surface at the point of incidence. (This straight line is called the "normal" to the surface at this point).

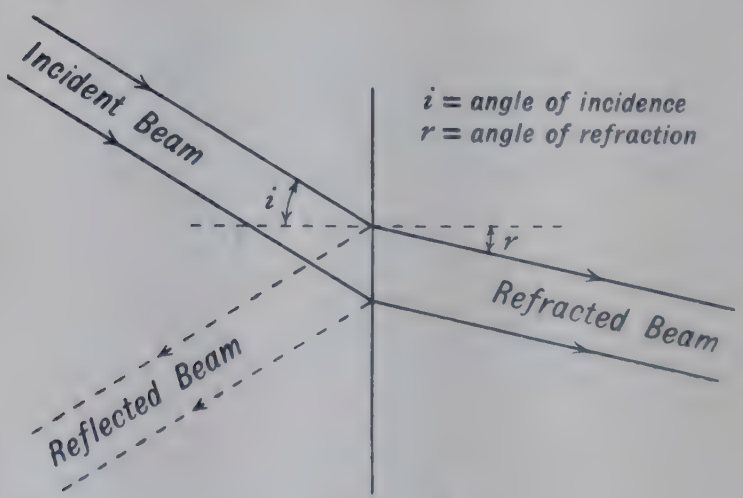

FIG. 26

(2) The sine of the angle of incidence (the angle between the incident ray and the normal) bears a constant ratio to the sine of the angle of refraction.

This ratio is constant for any particular pair of transparent substances with light of one particular colour. It is

called the ***refractive index*** of the second substance relative to the first.

i = angle of incidence.
r = angle of refraction.

Refractive indices of transparent substances are assumed to be measured relative to air unless it is specifically stated otherwise ; thus when we say that " the refractive index of water is 1·33 " we mean that for light passing *from air* to water

$$\frac{\sin i}{\sin r} = 1{\cdot}33$$

(for light of some particular colour).

In general for blue light the refractive index of a substance is larger than for red light. Some values of refractive indices are :

	blue	red
Water	1·34	1·33
Crown glass ...	1·53	1·51
Flint glass	1·64	1·60
Diamond	2·46	2·40

The refractive index of a substance is usually represented by μ.

Geometrical construction of the refracted ray

(1) The simplest method of tracing a refracted ray is to calculate the angle of refraction from the formula $\mu = \frac{\sin i}{\sin r}$. Then draw the angle r taking care that it is on the opposite side of the normal.

(2) A purely geometrical construction is sometimes preferred.
Suppose μ is 1·33.
Let QP be the incident ray. Fig. 27.
Mark a point A which is at a distance of 1·33 convenient units from the normal (so that NA = 1·33 inches, say). Draw a circle centre P and radius PA.
Find a point B on this circle which is on the far side of the normal and distant 1 unit from it. Then the refracted ray PR is drawn through B.

This is so because $\frac{\sin i}{\sin r} = \frac{NA}{PA} \Big/ \frac{MB}{PB} = \frac{NA}{MB}$ which by construction

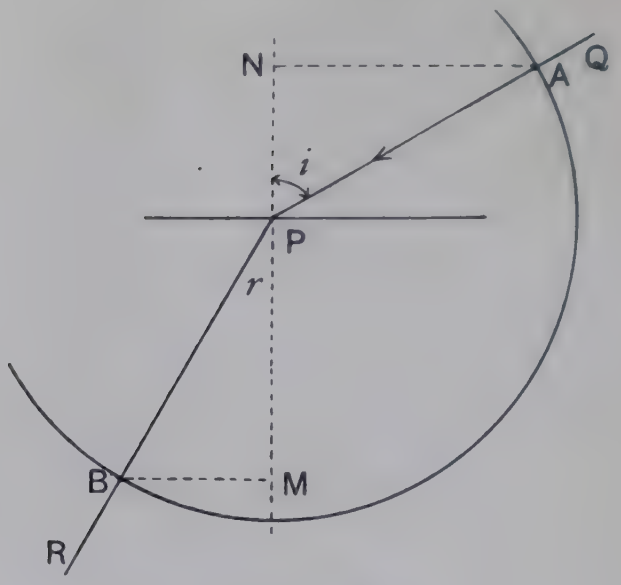

FIG. 27

has been made equal to 1·33.

If you use this construction you will not get accurate results unless you work carefully with a large circle and a fine pencil point.

A physical meaning of the refractive index

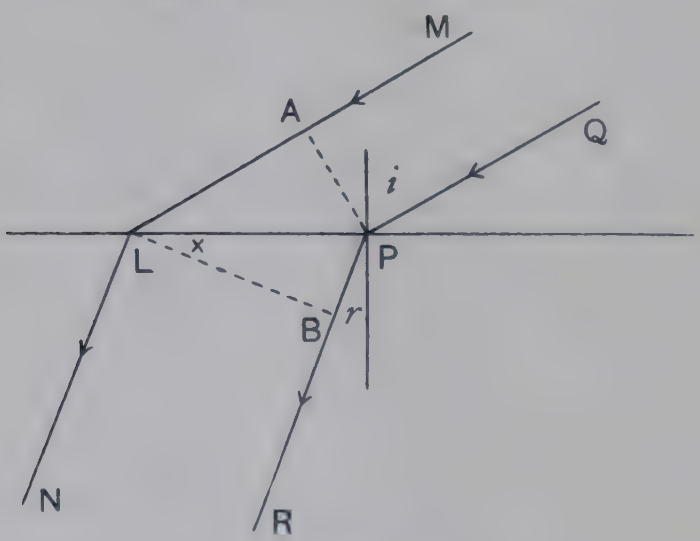

FIG. 28.

Suppose an incident beam of light changes its direction as shown in fig. 28.

Then $\mu = \frac{\sin i}{\sin r} = \frac{\sin APL}{\sin PLB} = \frac{AL}{BP}$

If now we suppose that light is some disturbance which travels with different speeds in different materials we see that one edge of the beam must travel along AL in the same time as the other edge is travelling, in the second substance, a distance PB.

Thus the physical meaning of the refractive index of the second medium relative to the first is

$$\mu = \frac{\text{speed of the light in first medium}}{\text{speed of the light in second medium}}$$

Thus if the refractive index of water relative to air ${}_a\mu_w$, is 1·33 we suppose that light travels 1·33 times as fast in air as in water.

This gives also a physical meaning to the relation

$$\text{Refractive index of glass relative to water } {}_w\mu_g = \frac{1}{{}_g\mu_w}$$

where ${}_g\mu_w$ is the refractive index of water relative to glass.

The critical angle

If a thin beam of light is made to fall at gradually increasing angles of incidence on an air to glass surface, the strength of the *reflected* beam is greater as the incidence becomes more oblique until, when the incidence is nearly 90°, very little light passes into the glass (fig. 29 (*a*)).

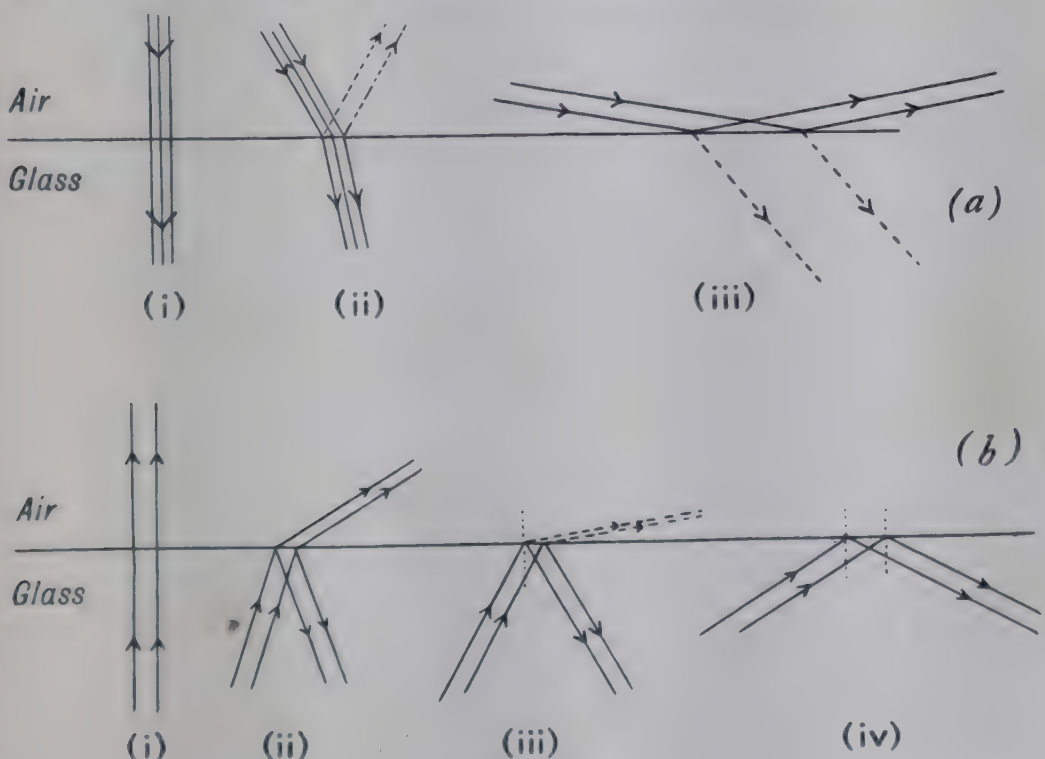

FIG. 29

Compare these results with those observed when light is falling on a glass to air surface (fig. 29 *b*). Here the angle of refraction into the air is greater than the angle of incidence in the glass. For small angles most of the light is refracted out into the air. As the incidence becomes more oblique the reflected beam becomes brighter and the refracted beam weaker.

At a certain ***critical angle*** (just bigger than in fig. 29 *b* (iii)) the refracted beam is very weak and the angle of refraction is 90° while almost all the light is reflected. For angles greater than this critical angle the glass-air surface behaves as a mirror and totally reflects all the light.

The value of the critical angle is given by

$$\frac{\text{sine of angle in air}}{\text{sine of angle in glass}} = {}_a\mu_g.$$

When the angle in the air becomes 90° its sine is 1.

Hence sine of critical angle in the glass $= \frac{1}{{}_a\mu_g}$

For glass of refractive index 1·50, the critical angle is therefore given by $\frac{1}{\sin C} = 1{\cdot}50$.

If you look the value up in a book of tables you will find that C is 41° 49′.

The image of an object immersed in water (fig. 30).

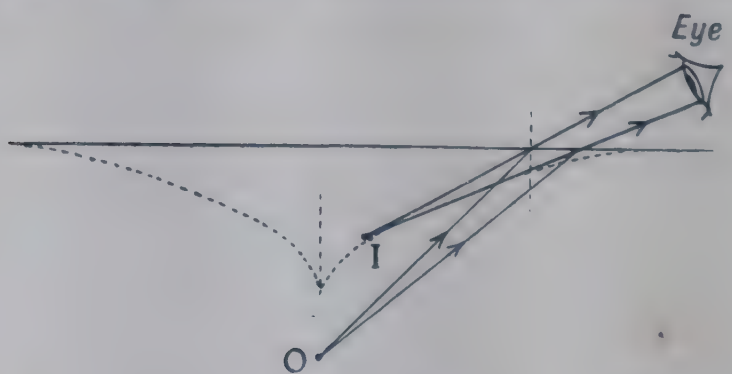

FIG. 30

To find the position of the image of O we must imagine a pencil of rays from O striking the water surface and being

refracted into the air. Such a pencil will appear to come from the image I, but careful investigation shows that the position of I is not the same for all emerging pencils but depends on the position of the eye.

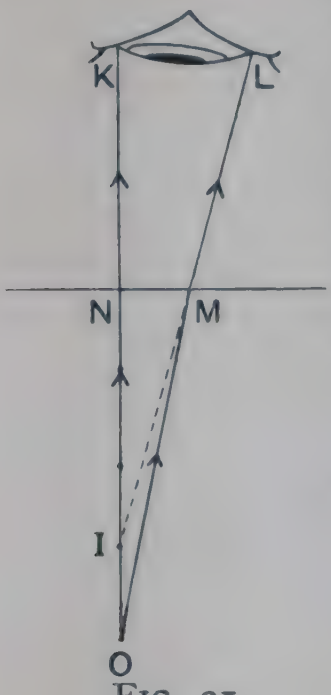

FIG. 31

When the eye is looking vertically downwards the problem is relatively simple (fig. 31). ONK represents a ray which striking the surface normally passes straight on.

OM represents a ray making an angle of incidence equal to NOM.

Hence

$$a^{\mu}w = \frac{\sin \text{angle in air}}{\sin \text{angle in water}} = \frac{4}{3}$$

$$= \frac{\sin \text{NIM}}{\sin \text{NOM}} = \frac{\text{NM/IM}}{\text{NM/OM}}$$

$$= \frac{\text{OM}}{\text{IM}}$$

Thus, *for nearly normal incidence,* $\frac{\text{ON}}{\text{IN}}$ is the same as $\frac{\text{OM}}{\text{IM}}$ and is very nearly $\frac{4}{3}$

$$\text{i.e. } \frac{\textbf{Real depth of object}}{\textbf{Apparent depth of image}} = \mu$$

For other positions of the eye it is well to remember that the image positions lie along the dotted curve shown in fig. 30. Such a curve is known as a "***caustic.***" Mirrors of very large aperture also give images along a caustic instead of a single image located at a definite point in space.

It is this apparent raising of objects seen in water which accounts for the apparent shallowness of a river and for the apparent crookedness of poles partly immersed in water. In all cases the effect depends partly on the position of the eye.

Refraction through a transparent slab

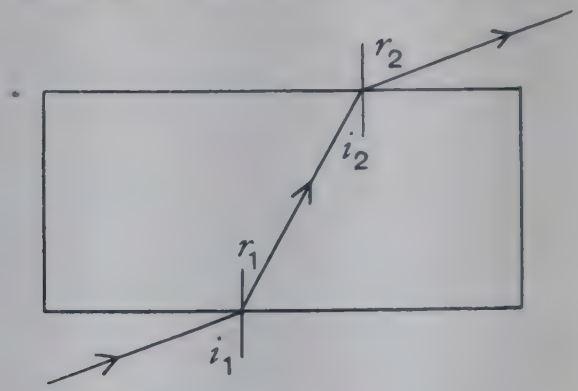

FIG. 32

If the light enters and leaves a transparent slab through plane surfaces which are parallel, experiment shows that the incident and emergent rays are parallel. Fig. 32.

This we should expect since the angle of incidence at the second surface is equal to the angle of refraction at the first.

Thus $\frac{\sin i_1}{\sin r_1} = {}_a\mu_g$ and $\frac{\sin i_2}{\sin r_2} = {}_g\mu_a$

Since ${}_a\mu_g = \frac{1}{{}_g\mu_a}$ and $\angle r_1 = \angle i_2$ it follows that $\angle i_1 = \angle r_2$. So that the emergent ray is parallel to the incident ray.

With a very thick slab of glass traces of colours may be observed in the emergent beam when using white light. These are explained later.

Refraction through a transparent prism

When the sides of the block are not parallel the emergent ray is not parallel to the incident ray. Fig. 33.

The angle which the emergent ray makes with the original incident ray is called the *angle of deviation* (D).

Experiments show that for any prism the amount of deviation varies with the direction at which the light meets the first surface.

If careful measurements are made and the deviation plotted against the first angle of incidence we get a graph (fig. 34) which shows a definite minimum. Experiment

shows that the minimum deviation occurs when the ray passes symmetrically through the prism making equal angles with the two faces. (Fig. 35.)

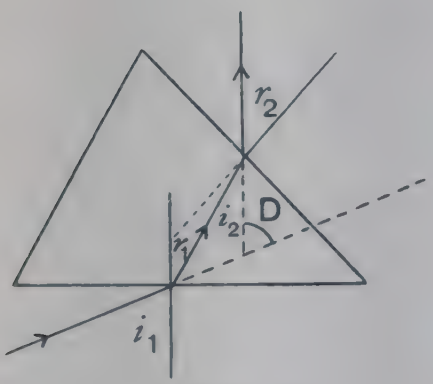

FIG. 33

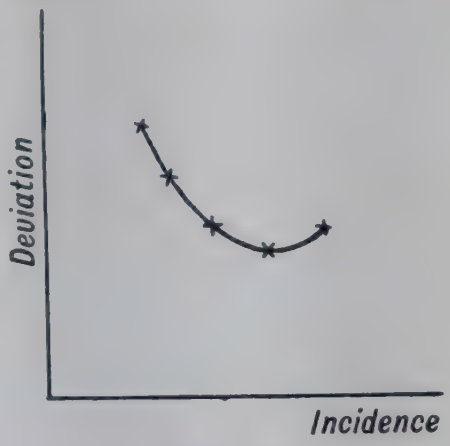

FIG. 34

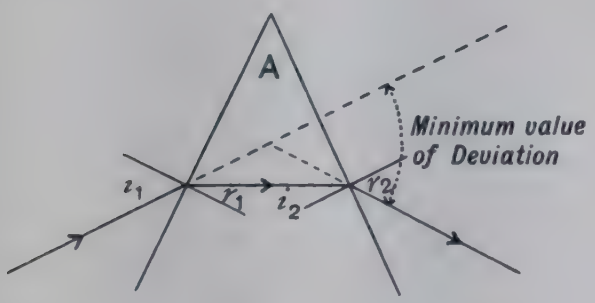

FIG. 35

When this is so it can be proved that if $\angle D$ is this minimum value and $\angle A$ is the "angle of the prism" (between the two faces used)

$$\mu = \frac{\sin\left(\frac{D + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

This formula is often used when a really accurate value for the refractive index of the prism is required.

Proof of the formula for minimum deviation

If it is assumed as an experimental fact that minimum deviation occurs when the ray makes equal angles with the faces of the prism,

then in the above figure $\angle i_1 = \angle r_2$ and $\angle r_1 = \angle i_2$

But by geometry $\angle A = \angle r_1 + \angle i_2 = 2\angle r_1$ and

$$\angle D = \angle i_1 - \angle r_1 + \angle r_2 - \angle i_2$$

Hence $$D + A = i_1 + r_2 = 2i_1$$

So that $$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin\left(\frac{D + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

The colours produced when white light passes through a prism

It has already been stated that the refractive index for any substance varies slightly with the colour of the light.

If a narrow beam of white light is sent through a prism, it is noticed that the edges of the emergent beam are coloured with " the colours of the rainbow."

This effect and numerous others explained by assuming that white light is a mixture of the many colours, can be seen in the rainbow and other so-called *spectra* of white light (see Chapter XX).

Total reflection in prisms

As the critical angle is less than 45° for all kinds of glass a very perfect mirror can be made by arranging that light falls on a glass-air surface at 45°. Such a mirror does not give the faint multiple images seen on looking obliquely at an ordinary plane mirror (see p. 139) and so total reflection prisms are used in prismatic field glasses and other instruments.

When it is desired to reflect the beam of light through 90°, a 45° prism is used as shown in fig. 36 (*a*). If the reflected beam is to be at 180° to the incident beam the same shaped prism is set as in fig. 36 (*b*).

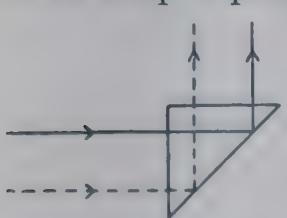

FIG. 36 (*a*)

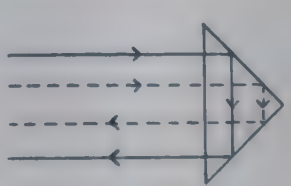

FIG. 36 (*b*)

FIG. 36 (*c*)

Total reflection at a layer of hot air (which has a lower refractive index than cold air) produces *mirages* which seem to show pools of water on roads or on the sands of the desert.

It is perhaps easier to think of such mirage effects as due to the change of direction of light waves due to the lower part of the wave front travelling faster than the upper part so that the wave front gradually alters its direction as shown by the dotted lines in fig. 37.

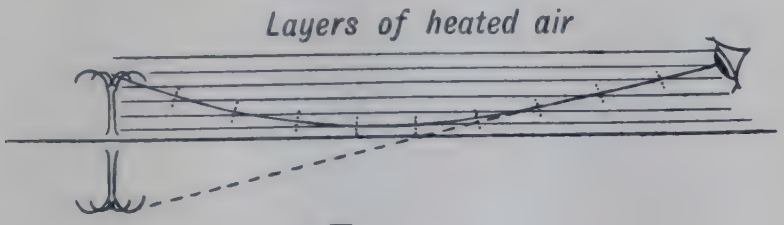

FIG. 37

Questions on Chapter XVII

(1) State the *laws of refraction of light.*

Describe how you would measure by an "apparent depth" method the refractive index of water contained in a beaker.

Prove the formula which you employ. (N.)

(2) Explain, with the help of diagrams, the formation of an image of an object by refraction at a plane surface separating two different media. Why does the position of such an image vary with the position of the observer?

Find the apparent depth of a pool of glycerine, the real depth of which is 3 in. The refractive index of glycerine is 1·47. Describe what you consider a good method of finding the refractive index of glycerine. (B.)

(3) Define "refractive index" of a substance.

Describe the method you would use and the calculations you would make to determine the refractive index of a transparent liquid. (L.)

(4) Explain why a ray of light travelling in water and striking the surface separating water from air at an angle of incidence greater than 49° cannot emerge into the air (μ for water = 1·33).

A fish is situated two feet below the surface of a pond. Describe, with the help of a careful diagram, what it will see in various directions, by direct vision and by reflection and refraction at the surface of the water. (C.)

(**5**) State the laws of refraction. A ray of light falls on one face of a parallel plate of glass and emerges from the opposite face. Find graphically or otherwise the displacement of the ray if its angle of incidence on the first face is 60°. The thickness of the block is 0·5 cm. and the index of refraction of the glass is 1·5. (O. & C.)

(**6**) Explain what is meant by *total internal reflection*. Under what conditions does it occur?

Show how a totally reflecting glass prism can be used instead of a plane mirror to turn a beam of light through a right angle. Explain why a prism of the same shape made of ice could not be used for this purpose. [Refractive index of glass = 1·5. Refractive index of ice = 1·3.] (O. & C.)

(**7**) A narrow parallel beam of white light is incident at 45° upon a rectangular block of red glass 5 cm. thick. Some of the light is reflected at the first surface; call this "beam *A*." The remainder passes into the glass at an angle of refraction of 30°, and falls upon the second surface of the block where some of it passes out into the air ("beam *B*"), and the remainder is reflected back to the first surface and passes out into the air; call this "beam *C*."

(*a*) Give a ray-diagram showing accurately the paths of the various beams.

(*b*) Remembering that red glass is being used, discuss the differences in colour between beams *A*, *B*, and *C*. (O.)

(**8**) A ray of light falls on one face of an equilateral glass prism at an angle of incidence of 20°. Make an accurate drawing to show the path of the ray through the prism, and determine the deviation produced. [Refractive index of glass of prism = $\frac{3}{2}$.] (C.)

(**9**) Describe, by means of a diagram, what is meant by the angle of minimum deviation for light passing through a prism.

Describe how you would measure the refractive index of the material of a given prism. (C.)

(**10**) When light has to be reflected and turned through an angle of 90°, a right-angled prism is often used instead of a silvered mirror. Explain with diagrams how this is done and why this method is preferred. Take the index of refraction for light passing from air to glass as 3/2. (D.)

(**11**) ABC is a glass prism, $AB = BC = 3$ in. and $\angle ABC = 90°$. Construct the path of a ray of light which passes through the prism, being incident on the face AB at a point 1 in. from A and parallel to AC. [Refractive index of glass $= \frac{3}{2}$.] Give a brief explanation of your construction and make your diagram full size. (L.)

(**12**) A glass sphere of radius 5 cm. has its centre at O. A ray of light starts from a point A and meets the sphere at P. If PA = 8 cm. and the angle PAO = 30°, trace the path of the ray by an accurate scale drawing, and explain the construction you use. [$\mu = 1{\cdot}5$.] (O.)

(**13**) Write an account of the apparent displacement of objects when they are viewed through the horizontal surface of water.

A small fish is swimming in a spherical bowl of 18 inch diameter. Where will it appear to be, if it is viewed horizontally, (*a*) when it is at the centre, (*b*) when it is moved towards the observer's eyes, to a position mid-way between the centre and the side? (A.)

(**14**) What do you understand by (*a*) *the refractive index* of a medium and (*b*) the *radius of curvature* of a lens face?

A bi-convex lens 3 cm. thick has faces of radii of curvature 10 cm. and 15 cm. respectively. Trace, on a good sized diagram, the course of a ray of light which comes from a point source placed on the axis of the lens 20 cm. in front of the face of smaller radius of curvature, and passes through the lens. The refractive index of the glass of the lens is 1·5. Explain your construction. (O. & C.)

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CHAPTER XVIII

REFRACTION AT SPHERICAL SURFACE

REFRACTION at transparent spherical surfaces takes place in accordance with the laws of refraction.

A lens which is thicker in the middle than at its edges is called a *convex lens* while one which is thicker at the edges is a *concave lens*. Lenses which have surfaces which are parts of spheres will, provided their aperture is not too large, give point images of a small source. The position of this image depends only on the particular lens used and the position of the object ; not on the position of the observer (see p. 167).

A line through the centres of the spheres which form the lens' surfaces is called ***the principal axis*** of the lens.

When sunlight falls on a convex lens in a direction parallel to the principal axis a real image of the sun is formed at a point called the ***principal focus*** of the lens. (See p. 167.) If the lens is turned round a real image will be formed on the other side. Thus the lens has two principal foci. Usually these are at the same distance from the lens. We shall also assume that our lenses are " thin " that is, that their thickness is small compared to their ***focal length*** (the distance from the lens to the principal focus).

The point at which the principal axis passes through the lens is called the ***optical centre*** of the lens. It is important not to confuse this optical centre with the centres of curvature of the two surfaces.

To measure the approximate value of the ***focal length of a convex lens*** catch on a screen the image of a distant object and measure how far this real image is from the lens. For the focal length of a concave lens, see page 173.

The paths of rays of light through a lens

With an apparatus which will produce a thin parallel beam of light, the following facts are easily verified :

(1) Any ray of light which falls on the optical centre of a lens passes through without change of direction. (Fig. 38*a*.)

(2) Any ray of light incident parallel to the principal axis of a lens passes through, or appears to come from, its principal focus after refraction.

With a convex lens, this focus is beyond the lens.

With a concave lens the light appears to come from its focus (a virtual one). (Fig. 38*b*.)

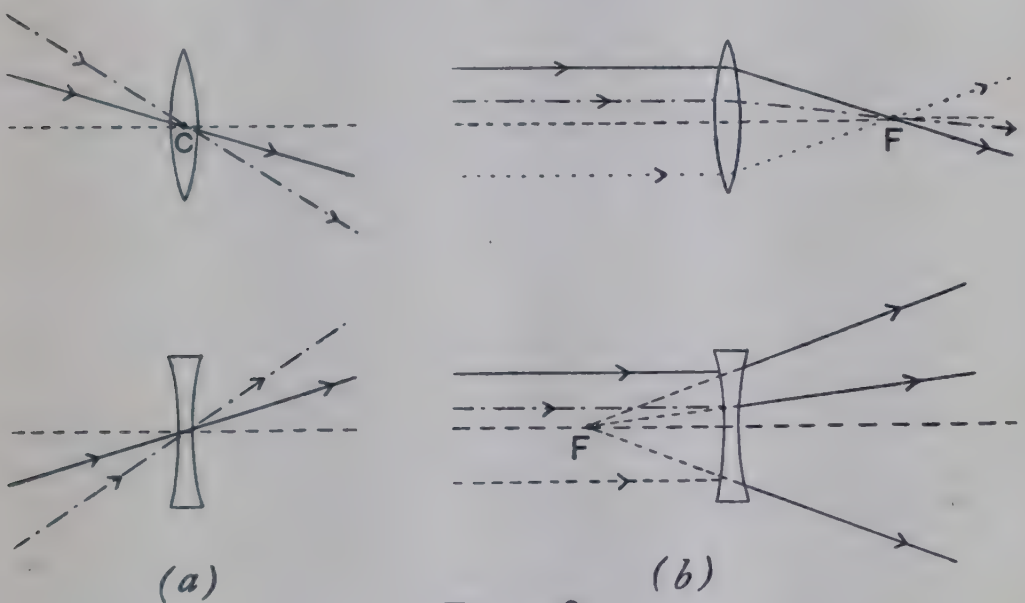


FIG. 38

A knowledge of these facts enables us to find, by a scale diagram, the position and size of the image of any object.

Geometrical method of finding the position of an image

Notice that in each diagram we have chosen two particular rays from the top of the object O and traced them through the lens.

The image of O is the point I through which the emergent rays pass (fig. 39*a*) or from which these emergent rays appear to come (fig. 39*b*). The points O and I are said to be *conjugate foci* or conjugate points ; since if light were directed towards I it would after refraction through the lens pass through O.

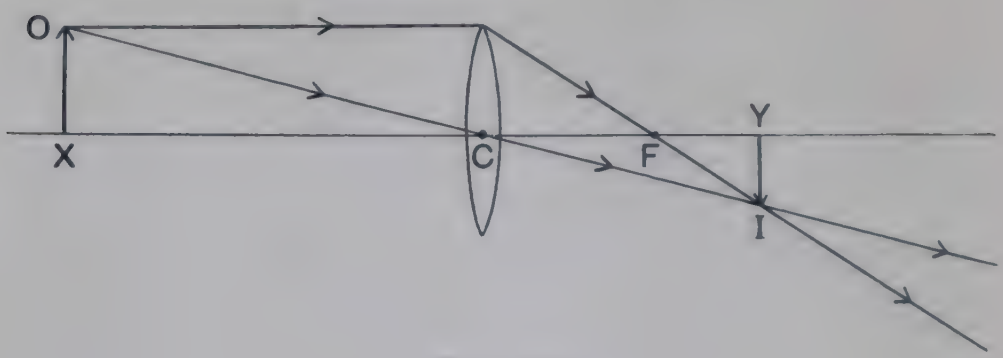
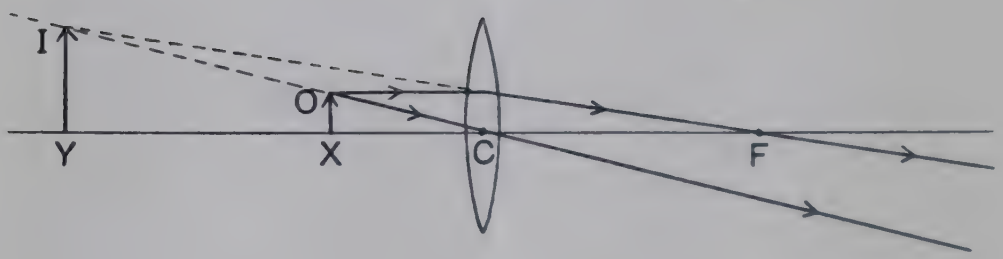


FIG. 39*a*



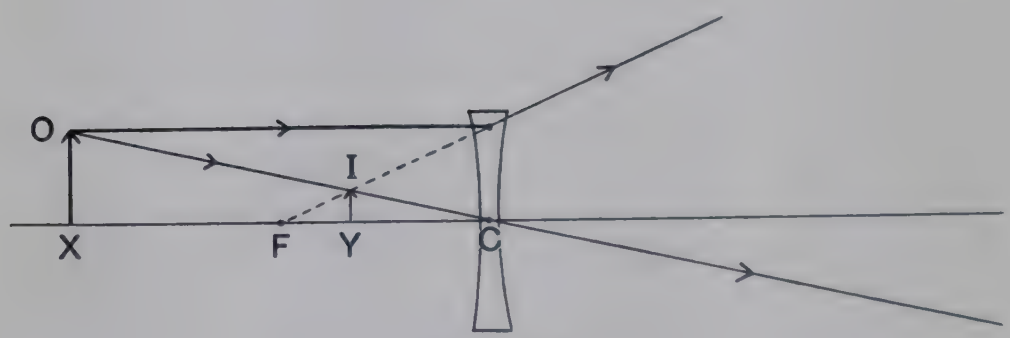


FIG. 39*b*

The pencil of rays by which an eye sees an image

Any ray from O will after striking the lens pass through I (or appear to come from I when the image is virtual).

The eye must be able to look at the image and at

FIG. 40

the lens. Thus the usual construction (p. 137) gives the space in which the eye can be put, and the pencil of rays can be drawn as in fig. 40.

The height of the image

Since the ray through the optical centre of the lens is not altered in direction but passes straight from the top of the object to the top of the image we have

$$\frac{\text{Height of image}}{\text{Height of object}} = \frac{\text{Distant of image}}{\text{Distant of object}}$$

Calculation of the distances of images from lenses

To save time it is usually better to make a rough ray diagram and then to calculate the position of the image from the equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (for proof see below)

Convention for positive and negative distances

Distances measured on the object side of the lens are taken as positive. Those on the other side are negative.

Another convention concerning the use of positive and negative signs is given on page 173.

The power of a lens

When a convex lens is used as a magnifying glass, the more powerful magnifier has the shorter focal length. Opticians speak of the power of a lens as defined by $\frac{1}{\text{focal length in metres}}$. Thus a convex lens of 20 cm. focal length is said to have a power of $\frac{1}{\cdot 2} = 5$ diopters.

If the above convention is used the power of a convex lens is a negative quantity because f is negative.

Opticians use the other convention (see page 173) and so speak of a convex lens as having a positive power.

Proof of the Lens Formula

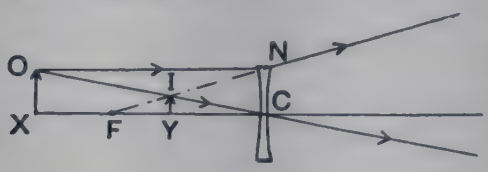


FIG. 41 (*a*)

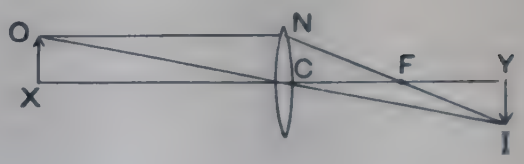


FIG. 41 (*b*)

Draw the usual rays OC, CI and ON, NI to locate the image IY. Then by the geometry of the figures 41 (*a*) and (*b*).

$$\frac{CY}{CX} = \frac{IY}{OX} = \frac{IY}{NC} = \frac{FY}{CF}.$$

For the concave lens fig. 41 (*a*)

$$\frac{CY}{CX} = \frac{FY}{CF}$$

becomes

$$\frac{v}{u} = \frac{f - v}{f}$$

$$\therefore\ vf = uf - vu$$

For the convex lens fig. 41 (*b*)

$$\frac{CY}{CX} = \frac{FY}{CF}$$

becomes

$$\frac{v}{u} = \frac{v - f}{-f}$$

because v and v — f are both negative while u is positive and f negative

$$\therefore\ -vf = uv - uf$$

Dividing through by v.f.u.

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Examples.

(1) It is required to project an image of an object, magnified threefold, on the screen 8 ft. from the object. What kind of lens must be employed, what must be its focal length and where must it be placed?

Magnification is 3

$$\therefore \frac{\text{Distance of image}}{\text{Distance of object}} = 3.$$

But total distance between object and image (screen) is 8 ft.

Hence distance of object from lens is 2 ft.
and distance of image from lens is 6 ft.

The image is on the opposite side of the lens to the object.
Hence $v = -6$.

$$\text{But } \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{-6} - \frac{1}{2} = -\frac{4}{6}$$

$f = -1\frac{1}{2}$ ft. the lens being convex.

(2) A concave lens of focal length 1 ft. produces an image which is $\frac{1}{4}$ as high as the object. Find the position of the object.

$$\text{magnification} = \tfrac{1}{4} = \frac{\text{Distance of image}}{\text{Distance of object}}$$

The lens is concave, hence the image is on the same side as the object and v is positive.

Hence $v = \frac{1}{4}u$

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{\frac{1}{4}u} - \frac{1}{u} = \frac{3}{u} = \frac{1}{f}$$

$\therefore u = 3f = 3$ ft.

N.B. In working out examples on mirrors and lenses it is advisable that the student should not prefix a sign to the *unknown* quantity, since the equation will give both the magnitude and *sign* of the unknown.

Experimental work with convex lenses

(1) The most important measurement for a convex lens is its focal length.

(*a*) This can be found quickly (to within a quarter of an inch) by catching on a sheet of paper, the inverted real image of a distant object.

(*b*) When a plane mirror is placed behind a convex lens and a bright object (lamp filament or pin) is placed at the focus of the lens an inverted real image of the same size as the object is formed at the same position as the object. This gives a simple and accurate measurement of the focal length.

(2) A bright object placed at a distance greater than the focal length will give a real image which can be caught on a screen.

This method enables us to measure (to $\frac{1}{10}$ inch) a number of corresponding values of v and u. Then by substituting in the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we can calculate values of f and find the mean value.

Experience shows that values of u equal approximately to 3f and 2f give the best results. An approximate value for f should always first be found by method 1 (*a*).

(3) Pin methods already described (p. 151) for concave mirrors are very accurate but need practice.

Remember that with a convex lens the image is real, inverted and diminished if the object lies at a distance greater than twice the focal length; real, inverted and magnified when the object distance is between 2f and f; and virtual, erect and magnified when the object is between the focus and the lens.

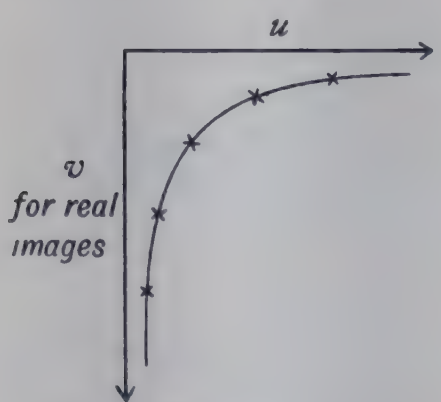


FIG. 42

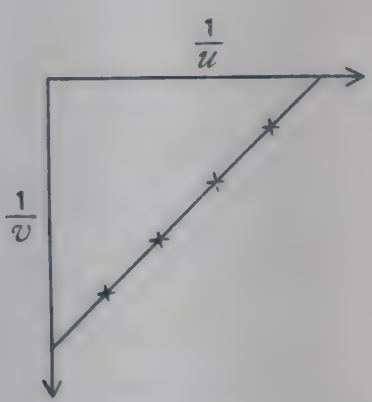


FIG. 43

Graphs of v and u for a convex lens appear as in fig. 42 if the same scale is chosen for v and u.

To get the value of f remember that when v is numerically equal to u it is also numerically equal to 2f.

Graphs of $\frac{1}{v}$ plotted against $\frac{1}{u}$ are straight lines which cut the axes at distances equal to 1/f (fig. 43).

Experimental work with concave lenses

The simplest way to find the focal length of a concave lens (f_1) is to put the lens in contact with a stronger convex lens (f_2) so that the combination is convex and of focal length F.

Then it can be shown that $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

(This formula applies to any two thin lenses in contact.)

Hence by measuring F and f_2 by either of the methods described for a convex lens, the value of f_1 can be calculated. Remember that the numerical values of F and f_2 will be negative for convex lenses on one convention but positive on the other (see below).

A method of measuring the radius of curvature of a convex mirror.

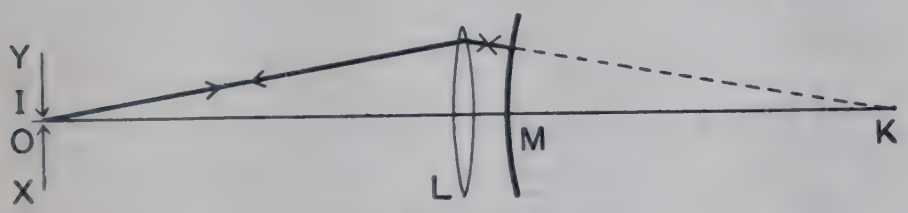


FIG. 44

If a convex lens L of focal length shorter than the radius of curvature of the mirror M is placed in front of the mirror it is possible to put an object OX in such a position that the real image IY is in contact with it. (The object can be the filament of an electric lamp or a pin.) Fig. 44.

When this adjustment has been made the light from O which passes through the lens must be falling on the mirror perpendicularly because the reflected beam is evidently returning along its own path to form the image at I. Hence if without moving L or the object we now remove the mirror and find the position of the real image formed by the lens alone, the distance MK is the radius of curvature of the convex mirror.

Another convention for the signs of distances

Another method of dealing with the signs of distances is to measure them from the mirror or lens surface and to call them positive if the light actually passes along the path and negative if the light only appears to come along the path.

Thus, on this convention, the distances of real images are always positive and those of virtual images negative.

This gives the same formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

for both mirrors and lenses.

It makes the focal lengths of concave mirrors and convex lenses (which have real foci) positive and those of convex mirrors and concave lenses (virtual foci) negative.

This method has the advantage that it gives similar signs to those used by opticians since they call convex lenses positive.

We will apply this convention to the examples already worked out for mirrors (page 150) and for lenses (page 170).

(1) An object 2 cm. high is placed 30 cm. from a concave mirror of focal length 10 cm. Calculate the position and size of the image.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{1}{v} + \frac{1}{30} = \frac{1}{10} \quad \therefore \frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30}$$

Hence $v = 15$ cm.

This result being positive, the image is real and inverted. It is therefore on the same side of the mirror as the object.

(2) An object is 10 in. from a convex mirror of radius of curvature 8 in. Find the position of the image.

A convex mirror has a virtual focus ; therefore the focal length in this case is -4 in.

$$\therefore \frac{1}{v} + \frac{1}{10} = -\frac{1}{4}$$

$$\therefore \frac{1}{v} = -\frac{1}{10} - \frac{1}{4} = -\frac{14}{40}.$$

$$\therefore v = -\frac{20}{7} = -2\tfrac{6}{7}.$$

Thus the image is virtual and $2\frac{6}{7}$ in. behind the mirror.

(3) It is required to project an image of an object magnified threefold on to a screen 8 ft. from the object. What kind of lens must be employed ; what must be its focal length, and where must it be placed ?

Magnification $= 3$

$$\therefore \frac{\text{Distance of image}}{\text{Distance of object}} = 3.$$

But total distance between object and image is 8 ft.

Hence distance of object from lens is 2 ft.

and distance of image from lens is 6 ft.

The image projected on a screen must be real

$\therefore$ v is positive

But $$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\therefore \frac{1}{f} = \frac{1}{6} + \frac{1}{2} = \frac{4}{6}$$

$\therefore f = 1\frac{1}{2}$ ft. the lens being convex.

(4) A concave lens of focal length 1 ft. produces an image which is $\frac{1}{4}$ as high as the object. Find the position of the object.

$$\text{Magnification} = \tfrac{1}{4} = \frac{\text{Distance of image}}{\text{Distance of object}}$$

The lens is concave, hence its focal length is negative, and the image is virtual (v is negative).

$\therefore v = -\frac{1}{4}u$ and $f = -1$

But $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{1}{-\frac{1}{4}u} + \frac{1}{u} = \frac{1}{-1}$$

$$\therefore -\frac{4}{u} + \frac{1}{u} = -1$$

$$\therefore -\frac{3}{u} = -1 \quad \therefore u = 3 \text{ ft.}$$

Questions on Chapter XVIII

(1) State clearly what is meant by the *focal length* of a convex lens, and describe one good method of measuring it.

A bright object 12 in. from such a lens gives a real image on a screen 8 in. beyond the lens. The object is moved 6 in. nearer to the lens. How must the screen now be moved so as to receive a clear image? (N.)

(2) What do you understand by the *focal length* and the *optical centre* of a lens?

Prove, for a convex lens, that, if u and v are the distances of the object and image respectively from the lens, the magnification produced is v/u.

An object 5 ft. high and 40 ft. distant from the camera lens is photographed with a camera having a lens of focal length 8 ins. What is the height of the image? (O. & C.)

(**3**) What do you understand by *conjugate foci* of a lens?

Prove that, if v and u are the distances of image and object respectively from a lens of focal length f, then $1/v - 1/u = 1/f$.

A lantern lens is used to project an image of a $2\frac{1}{2}$ in. square lantern slide on to a screen 20 ft. away from the lens. The image is to be 5 ft. square. What must be (*a*) the distance of the slide from the lens, (*b*) the focal length of the lens? (O. & C.)

(**4**) Explain, giving diagrams, how a convex lens can be made to give (*a*) a virtual, (*b*) a real, image of a source of light. How would you find the focal length of such a lens?

Explain why a convex lens can be used as a burning glass. (O.)

(**5**) An object 4 cm. long is placed 100 cm. in front of a convex lens of focal length 20 cm. and perpendicular to the principal axis of the lens. What is the position, nature, and size of the image produced?

What would be the nature of the image if the object were 10 cm. in front of the lens? (N.)

(**6**) A convex lens of 20 in. focal length forms an image of an arrow which lies along the axis of the lens with its mid-point 30 in. from the lens.

The length of the arrow is 2 in. Find the length of the image. (O. & C.)

(**7**) Find the position, size and character of the image produced by a convex lens of focal length 7 inches of an object 2 inches tall placed (*a*) 4 inches, (*b*) 10 inches from the lens. Draw a diagram showing the paths of two rays in each case. (L.)

(**8**) Make a drawing to scale (half size) of the path of a

pencil of rays falling on a convex lens of 10 cm. focal length—

(*a*) when the pencil diverges from a point on the axis of the lens 25 cm. from it ;

(*b*) when the pencil is a beam parallel to the axis ;

(*c*) when the pencil converges towards a point on the axis 25 cm. beyond the lens. (W.)

(**9**) An object is placed perpendicular to the axis of a convex lens of 10 inches focal length and 6 inches from the lens. Show by a diagram the paths of the rays by which the object is seen by an eye on the axis of the lens and 20 inches from it. (L.)

(**10**) An object is placed 10 cm. from (*a*) a converging lens of 20 cm. focal length, (*b*) a diverging lens of 20 cm. focal length. Calculate the position of the image in each case, and draw diagrams to show how the image is formed. (O. & C.)

(**11**) Distinguish between *real* and *virtual* images, illustrating your answer by means of diagrams.

An object of height 1 cm. is placed at a distance of 20 cm. from a thin lens, and the image is found to be 3 cm. high and real.

What kind of lens is it, and what is its focal length ? (C.)

(**12**) You are given a lens and asked to find out whether it is a convex or a concave one. Explain how you would do so.

Describe, in detail, how you would measure its focal length if it were a weak concave lens. (C.)

CHAPTER XIX

OPTICAL INSTRUMENTS

The Camera

FIG. 45

THE camera consists of a light-tight enclosure, usually with adjustable sides B, in the front of which is a convex lens C held parallel to a plate or film P. A shutter is used to close the lens or to make the exposure and there is usually a diaphragm or stop which is adjustable so as to allow more or less light to pass through the lens.

The distance of the lens from the plate has to be adjusted so that the image IY (fig. 45) is formed exactly on the sensitive surface of the plate. If this is not done, the picture will be " out of focus." Experience shows that the accuracy of focusing required depends mainly on the effective aperture of the diaphragm of the lens. If this is large (f/4·5) exact focusing is necessary. If it is about $\frac{1}{16}$th of the focal length of the lens (f/16) all objects from 15 feet to infinity will be more or less in focus at the same time and so a cheap camera can be made without any focusing arrangements.

Expensive cameras have convex lenses made of a number of convex and concave lenses arranged to avoid the distortions and colour effects which are produced by a simple lens.

The optical or enlarging lantern

A lantern intended to throw on a screen a bright image of a photograph has a convex projection lens and some

arrangement for passing a strong beam of light through the projector after the light has strongly illuminated the photograph.

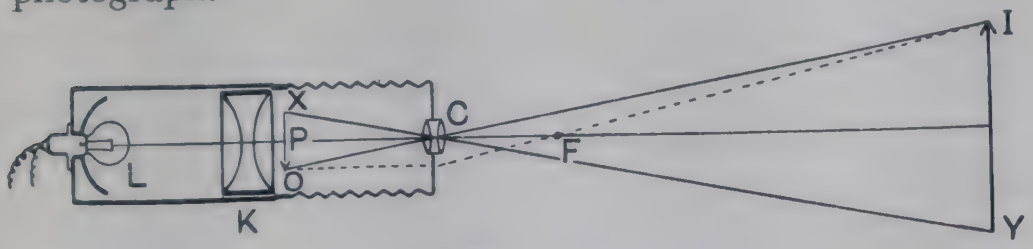


FIG. 46

Light from the lamp L is focused by a large "condenser" K so that the image of the lamp is formed in the opening of the projection lens C. The photograph P is just in front of the condenser so that the light passes through it. The convex projection lens, usually a compound lens to avoid distortion and colour effects, forms an enlarged, real, inverted image of the plate on the screen S. In order that the final image IY may be the right way up the lantern slide OX must be put in upside down.

The Eye

In its optical construction the eye of man and of most animals resembles a camera.

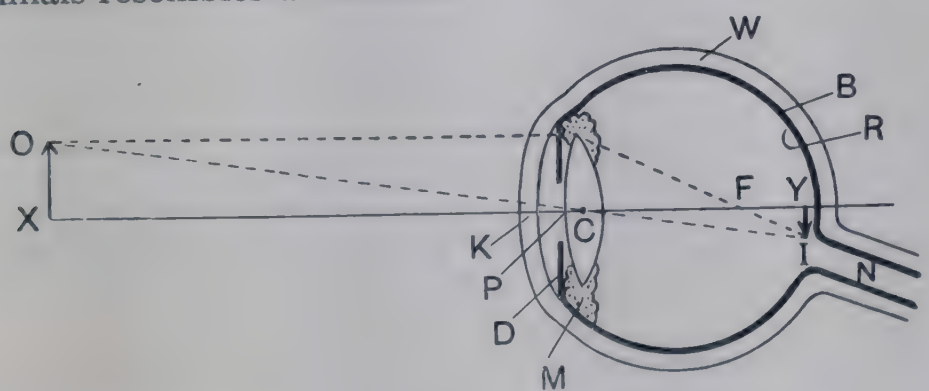


FIG. 47

The "white" of the eye forms the outer layer which becomes transparent in front where it is known as the cornea.

Under this layer is an opaque coat B which behind the cornea forms a coloured diaphragm the iris of the eye. A

hole in the centre of this, the pupil, allows light to fall on a convex lens (C), the crystalline lens. This acts like the lens of a camera and produces a small real, inverted image IY on a sensitive layer called the retina (R), an extension over the interior of the eye of the optic nerve N which carries the visual sensations to the brain. The brain in some way learns to see the object the right way up when the image on the retina is inverted.

In focusing the image IY sharply on the retina the eye, unlike the camera, varies the curvature of the surfaces of the crystalline lens by means of the ciliary muscles M. This focusing of the eye is known as the ***"power of accommodation"*** and a normal eye can form a sharp image of objects which lie between infinity and a distance of about 8 inches. When an eye can only see clearly objects from, say, 4 inches to 4 feet, it is said to be short sighted. An eye which sees distant objects clearly but cannot, without great effort, see clearly objects which are near is long sighted. In old age, people often lose the power of accommodation and can only see clearly over a narrow range say from 3 feet to 10 feet.

The distance of most distinct vision for a normal eye is about 10 inches.

Another defect of the eye is astigmatism. When a person with this defect looks at say a brick wall, he notices that when the horizontal lines on the wall appear sharply defined the vertical ones are blurred, and *vice versa*.

A rarer defect is colour blindness. About 4% of men and considerably less women have difficulty in distinguishing certain colours particularly dull browns and greens. This is said to be due to the absence of certain cells in the retina.

The use of spectacles

The defects of sight, other than colour blindness, can be remedied by the use of lenses placed in front of the defective eye.

Short sight

The short sighted eye has its distance of most distinct vision about 6 inches and its far point at, say, 4 feet. It will therefore need the assistance of a *concave* lens of focal

length 4 feet so that a distant object seen through the spectacle lens will appear at the far point of the eye, where it can be seen without effort. Such a lens will also make near objects appear nearer the eye, but as the eye can see things very near to it, this will not be a disadvantage.

Long sight

A long sighted eye can see the distant objects but will need assistance when reading. Hence it should have a lens which will make an object at, say, 1 foot appear to be at its near point which is perhaps 3 feet away.

Hence for the spectacle lens

$$\frac{1}{3} - \frac{1}{1} = \frac{1}{f} \qquad f = -1\tfrac{1}{2} \text{ feet}$$

Hence a convex lens of focal length 18 inches is required.

On the other convention (p. 173) the calculation will be

$$\frac{1}{-3} + \frac{1}{1} = \frac{1}{f}$$

since the image is virtual. Hence $f = 1\frac{1}{2}$ feet.

Hence a convex lens of focal length 18 inches is required.

Loss of accommodating power

When the eye can see clearly only over a limited range, say, from 3 feet to 10 feet, it will need two lenses.

To read a book at 1 foot it requires a *convex* lens of focal length 18 inches (given by the equation above). Distant objects must be brought to 10 feet and so a *concave* lens of focal length 10 feet will be necessary. The lenses are often fitted together to form a " bifocal " lens, the top part for viewing distant objects and the lower part for reading.

Astigmatism is corrected by the use of lenses which have different curvature in different directions.

The magnifying glass or simple microscope

When you wish to see a small object most clearly you bring it to the distance of most distinct vision (D) which for a normal eye is about 10 inches. A short focus convex lens placed near the eye will enable you to bring the object much nearer because the lens will form a magnified, erect, virtual image which can be brought to the 10 inch distance. Such an arrangement forms a simple microscope (fig. 48).

The magnifying power of an instrument (as distinct from the magnification of an image) ***is generally defined as the ratio of the angle at which the final image is observed to the angle at which the object can be observed distinctly with the unaided eye.***

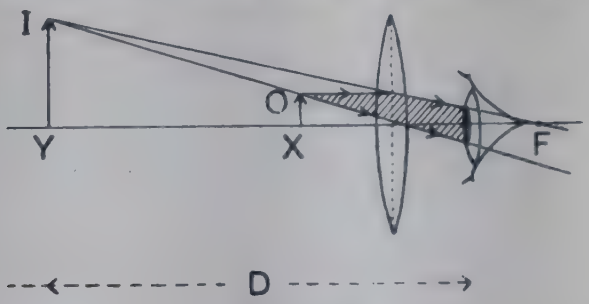


FIG. 48

From the above diagram, the angle of the image is evidently $\frac{IY}{D} = \frac{OX}{f}$

If the object were observed unaided it would have to be held at D from the eye so that its angle would then be $\frac{OX}{D}$.

Hence the magnifying power of the simple microscope is D/f which becomes larger as f becomes smaller.

The compound microscope

The simplest form of compound microscope has two lenses ; a short focus convex objective and a short-focus convex eyepiece (fig. 49).

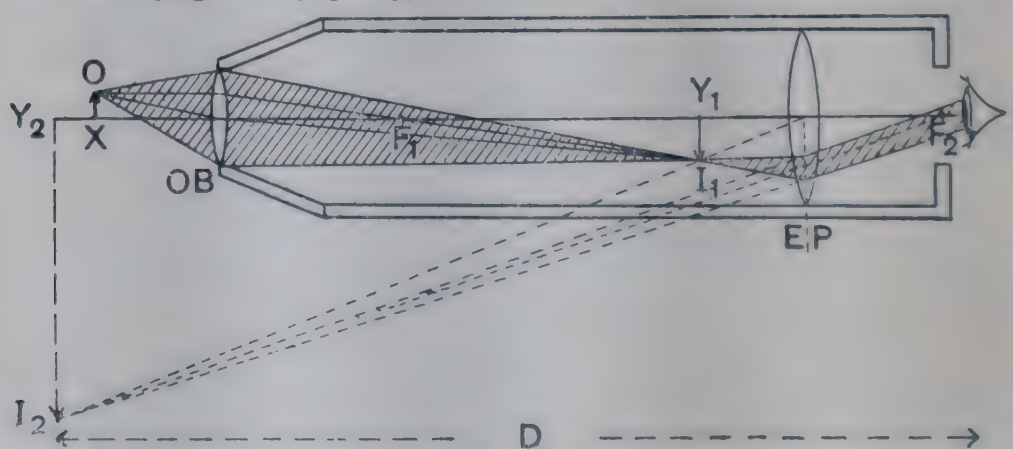


FIG. 49

The objective OB is moved to such a distance from the object OX that it gives an enlarged real image I_1Y_1.

The eyepiece EP is used as a magnifying glass to give a virtual magnified image of I_1Y_1 often at the distance D from the eye.

The figure shows that a pencil of light from O through the objective crosses the axis at some distance from the eye piece lens. A small opening, the eye ring, is generally put above the eyepiece in order that the eye of the observer shall be in the best position to see the final image. The magnifying power increases with shortness of the focal lengths of both the objective and the eyepiece.

The Astronomical Telescope

A simple astronomical telescope has a long focus convex objective and a short focus convex eyepiece. (Fig. 50.)

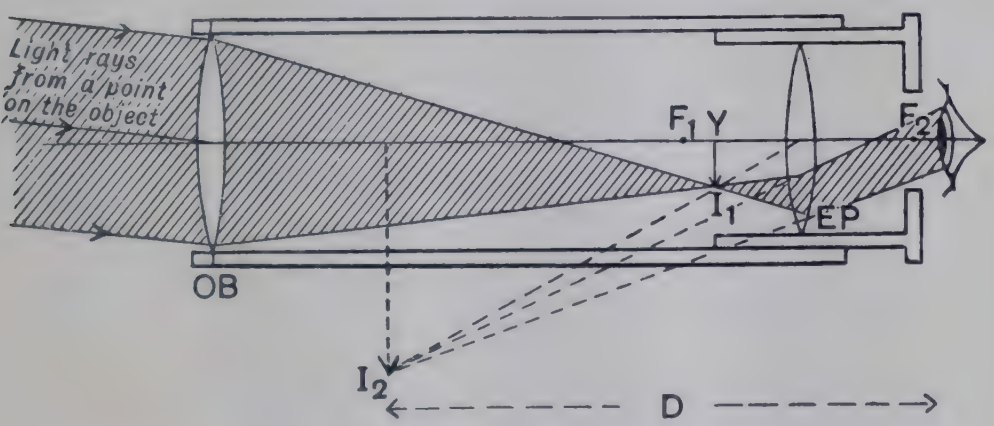


FIG. 50

The objective OB, preferably large to gather as much light as possible, gives a real inverted image very near its focus F_1.

The eyepiece EP is used as a magnifying glass to give a virtual magnified image of I_1Y. Some eyes focus the telescope to give this image at the distance D ; others focus it so that parallel rays emerge from the eyepiece, that is, so that the final image is at infinity. In the latter case, it can be shown that the magnifying power of the telescope is

$$\frac{\text{focal length of objective}}{\text{focal length of eyepiece}}$$

The Field Telescope

The astronomical telescope gives its final image upside down. In a field telescope the final image must be upright and so an additional short focus convex erecting lens has to be used. The simplest form with 3 convex lenses is shown diagramatically. (Fig. 51.)

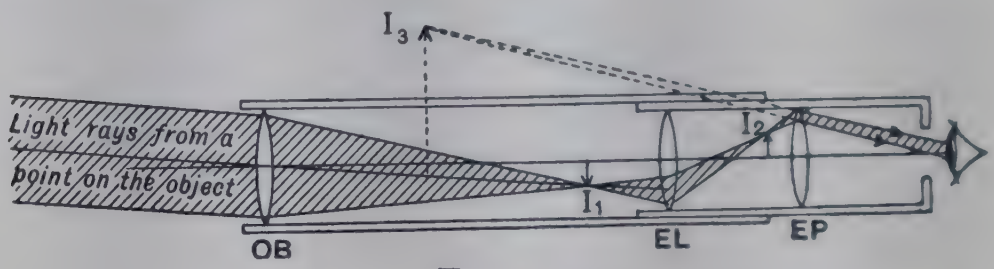


FIG. 51

In actual microscopes and telescopes all the lenses are compound lenses to reduce colour effects (see p. 191) and distortions.

The Galilean Telescope

Galileo invented a telescope which is still used in opera glasses and some forms of " field-glasses."

It has a convex objective and a concave eyepiece. The path of a pencil of light through it is illustrated in fig. 52.

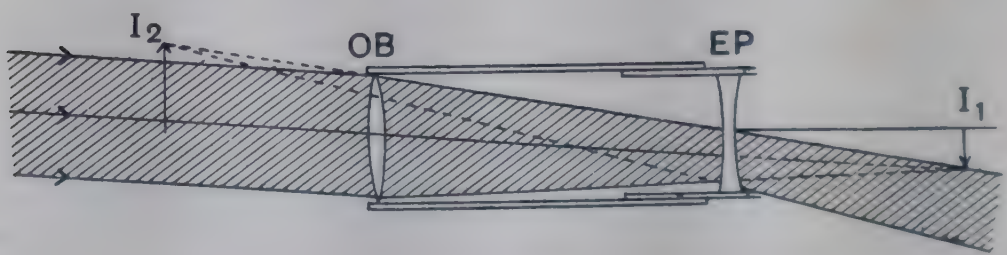


FIG. 52

The objective would by itself produce a real image I_1 but the eyepiece changes the direction of the light so that, to an eye placed directly behind the eyepiece, the light appears to be coming from a virtual image I_2 which may be at the distance of most distinct vision or may be at infinity.

This form of telescope gives upright images and it has a

shorter tube for a given magnifying power than the other form has.

Its disadvantage is that its field of view is smaller than that obtained by the other form of telescope.

Questions on Chapter XIX

(**1**) Explain, with the help of a diagram, the construction of a single lens camera.

Using a given camera it was found that in order to take a photograph of a distant object the photographic plate had to be at a distance of $7\frac{1}{2}$ in. from the lens. How far must the plate be from the lens when the object is only 10 ft. in front of the lens, and what will be the length of the image on the plate if the object is 3 ft. high ? (N.)

(**2**) Explain with the aid of a diagram the optical principles involved in the projecting lantern. Explain clearly the necessity for the condensing lens.

A lantern has a focusing lens of focal length 11 inches, and the distance of the slide from this lens is 12 inches. Find how far the screen must be from the focusing lens. If the slide is a square of side 3 inches, find the area of the image. (O.)

(**3**) It is required to project a lantern slide from a lantern at the back of a hall on to a screen 20 feet square. The slide is 4 inches square and the hall is 100 feet long. You are provided with two projecting lenses, one of focal length 20 in. and the other of focal length 15 in. Find which lens would be the more suitable, giving reasons for your choice. (C.)

(**4**) Describe the optical system of the eye, and draw a diagram to illustrate the formation of an image on the retina.

A short-sighted person can see distinctly objects which are not more than 12 in. from the eye. What kind of lens should he use, and of what focal length to enable him to see clearly at 20 in. ? (O.)

(**5**) What is meant by the statements that an eye is (*a*) long-sighted, (*b*) short-sighted ?

A man cannot see objects clearly at a distance greater than 3 ft. Explain with the aid of careful diagrams how he can be enabled to see distant objects clearly, and state the nature and focal length of the glasses required. (C.)

(**6**) Explain the defects of vision known as " short sight " and " astigmatism," and show how they may be corrected by the use of suitable lenses.

The least distance at which a person can see objects clearly is 100 cm. What type of lens must he use to be able to see objects distinctly which are distant 25 cm. from him and what will be its focal length? (O. & C.)

(**7**) A long-sighted person requires spectacles. He needs a lens which will form, at a distance of 30 in. from it, an erect image of an object which is 10 in. from it. Find the focal length of the lens required.

Show by a diagram the path of a pencil of rays from a point of the object not on the principal axis. (W.)

(**8**) Describe briefly the optical arrangements of the eye and explain how the eye adjusts itself to see objects at different distances.

Explain with the help of a careful diagram how a convex lens of short focal length, *e.g.* 2 in., held close to the eye, can be used as a magnifying glass. Why does the arrangement make a small object appear larger than it could appear to the unaided eye? (W.)

(**9**) Explain the action of the simple magnifying glass.

Where must a lens of 5 cm. focal length be placed in front of a small object so as to form an image 25 cm. from the lens on the same side as the object? What is the magnification? (W.)

(**10**) A convex lens is to be used as a magnifying-glass by a person whose nearest distance of distinct vision is 12 in., and a magnification of 3 is to be produced. What must be the focal length of the lens?

Illustrate your answer by a diagram to scale. (B.)

(**11**) Explain the construction of a compound microscope.

On a large-scale diagram show the course of a set of rays through the optical system indicating the positions of the

object and of the images formed. Are the images real or virtual?

Describe clearly, with reasons, the effect on the magnifying power of the microscope of increasing the distance of the eyepiece from the object glass. (O.)

(**12**) Describe the Galilean telescope (converging object-glass and diverging eyepiece) and illustrate your answer by means of a diagram. (O. & C.)

(**13**) Describe the construction of a simple astronomical telescope and show how the image is formed.

A telescope, fitted with cross-wires, is mounted so that it can be rotated about a vertical axis. It stands in a room with an open window through which it is focused on a distant object, A, and then on another B. The angle through which the telescope has to be turned is measured on a circular scale. The observations are repeated after closing the window which is glazed with $\frac{1}{4}$-inch plate glass. What will you expect to find? Give reasons. (A.)

(**14**) Two biconvex lenses of focal lengths 4 cm. and 20 cm. are to be combined to form (i) an astronomical telescope, (ii) a microscope. Give diagrams for each case, showing how you would place the lenses. Mark the object glass and eyepiece O and E respectively.

The astronomical telescope is adjusted to view a distant object and is then turned round so that the object glass is next to the eye and the same object is again observed. Explain whether there will be any difference in the images seen in the two cases. (D.)

(**15**) Explain, with the aid of diagrams, the optical action of (*a*) a simple magnifying glass, and (*b*) a simple terrestrial telescope. Why is the diameter of the object glass of a telescope always greater than that of the eye-lens? (A.)

CHAPTER XX

SPECTRA AND COLOUR

THE refractive index of transparent substances varies slightly with the colour of the light used. Thus for ordinary glass, measurements with violet light show $\frac{\sin i}{\sin r} = 1{\cdot}53$ while for red light the value of the refractive index is 1·51 (see p. 155).

Careful observation of a narrow beam of light passing from air into glass shows that there is a slight separation of rays of different colours. This is shown diagrammatically in exaggerated form in fig. 53.

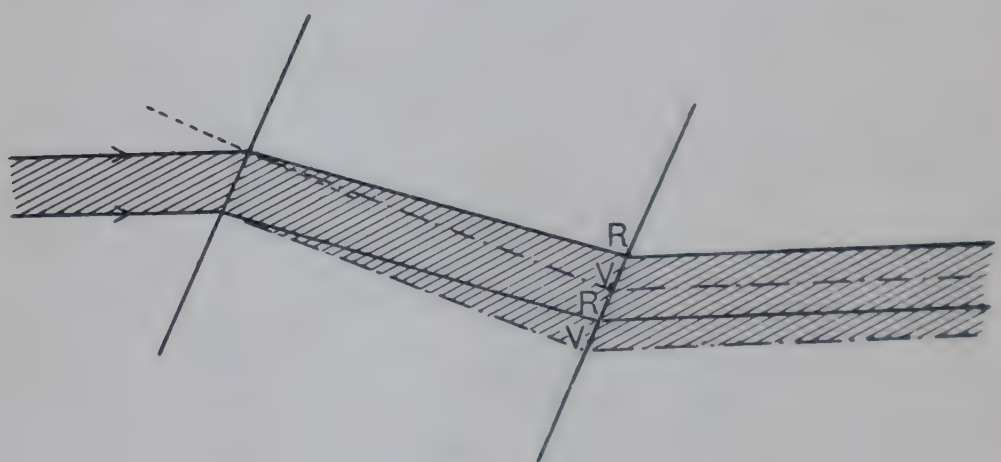


FIG. 53

With a slab of glass having parallel faces, the emergent beams of red and of violet light and of all the intermediate colours are all parallel to the incident beam and since the separation produced by a thickness of a few inches of glass is small, we rarely observe any colour effects.

With a glass prism (fig. 54) the emergent beams are diverging from each other and so the angular separation of the colours is at once evident, the violet being deviated through a greater angle than the red, while the other colours appear between these two extremes.

Here even with a wide incident beam we observe colours near the edges of the emergent beam, bluish on one side and red on the other.

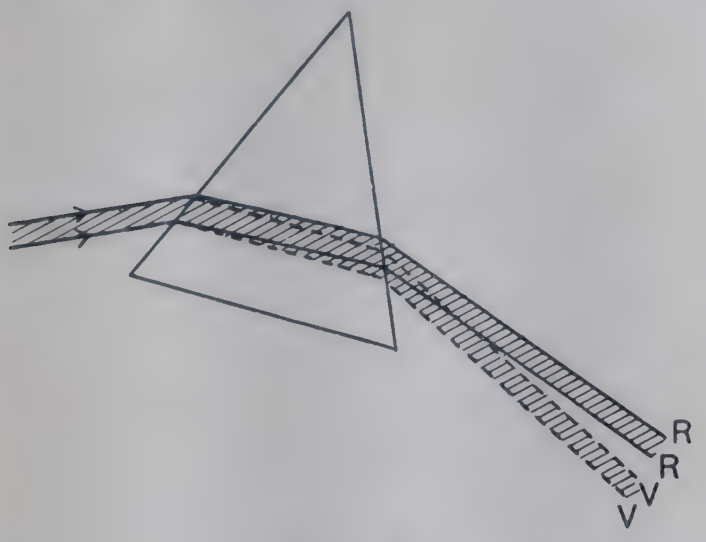


FIG. 54

If the light from a slit after being dispersed by a prism is allowed to fall on a white screen it forms a set of overlapping patches of colour which form a very impure or mixed ***spectrum*** because in the middle all the colours overlap and give white light while at one edge red appears and at the other violet.

If the eye is placed behind the prism so as to look at a fine slit from which white light is diverging the eye sees the slit drawn out into a spectrum.

In this spectrum the colours are not mixed because the eye focuses each diverging pencil of coloured light and so sees a virtual image of the corresponding colour as shown in fig. 55.

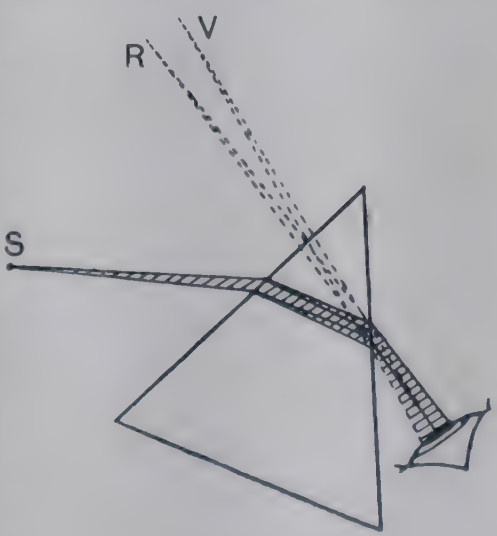


FIG. 55

Rainbows are formed by the dispersion of the various colours of sunlight which has passed through drops of water.

The phenomena are com-

plicated and it will be enough to say that the primary (inner) bow is produced by rays which have been reflected once and refracted as shown at P in fig. 56. The secondary bow S is formed by two reflections within the raindrop.

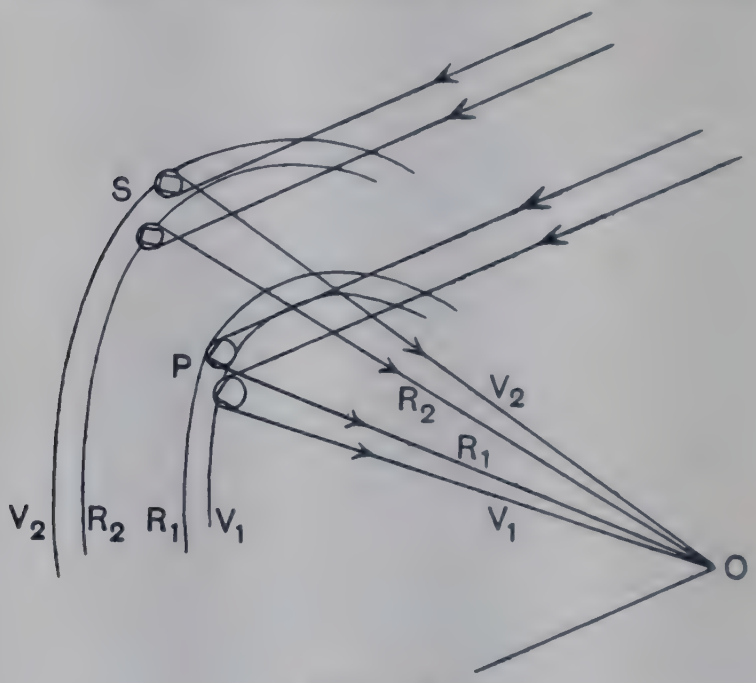


FIG. 56

In order that emergent rays shall reach the eye of an observer at O the drops of water must lie on the circles shown in perspective.

The projection of a pure spectrum

In order to project a pure spectrum a lens must be used to focus the diverging pencils of coloured light as they emerge from the prism. A strong source of light is usually placed behind the slit S. (Fig. 57.) The size of the spectrum on the screen depends partly on the angular separation of colours produced by the prism and partly on the distance of the final real image from the lens.

(The lens can also be placed on the other side of the prism to make the light convergent before it enters the prism.)

The reason for the use of a slit is that the spectrum is a set of images of the slit. If it is narrow and rectangular there will be little overlapping of the coloured images.

If it were large and round, the images would be of corresponding size and a whitish centre with coloured edges would be seen on the screen. This would be a very impure spectrum.

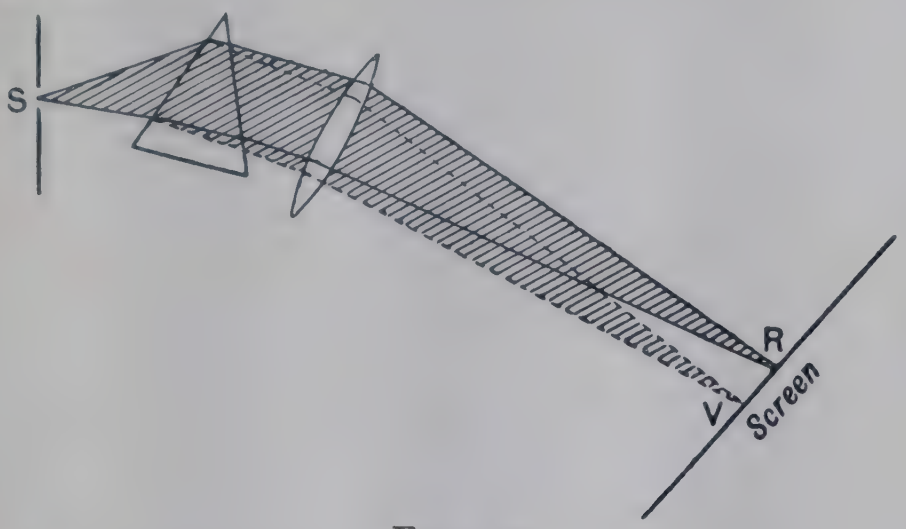


FIG. 57

The Prism Spectroscope

In a spectroscope there is usually a more complicated system of lenses.

The eyepiece magnifies the small spectrum which can be seen in the tube if the eyepiece is removed.

Dispersion produced by a lens

FIG. 58

The passage of white light through a lens is accompanied by dispersion so that the violet is brought to a focus nearer to the lens than is the red light (fig. 58).

This effect is very noticeable when simple telescopes are made by using a thin lens as objective It is called " chromatic aberration."

To remedy it, a compound lens is designed so as to be

" achromatic " and give images which are practically colourless. The simplest combination is a convex lens of crown glass (refractive index 1·53 to 1·51) in contact with a weaker concave lens of flint glass (refractive index 1·64 to 1·60). The resultant lens is convex but the separation of the colours produced by the convex part is just compensated by the dispersion in the other direction due to the concave lens. Thus the addition of the flint glass concave lens destroys the " dispersion " but not all the " deviation " so that the combination still has some convergence.

Types of Spectra

The spectrum produced by a heated solid or liquid is ***" continuous,"*** that is, it shows a band in which the colours merge into each other. The spectrum of a filament electric lamp whose temperature can be altered by means of a suitable resistance, shows at low temperatures strong red with little violet while as the temperature is raised the violet becomes more prominent.

Incandescent gases—such as emit the light in neon and other luminous signs, show a ***line spectrum***—the light when analysed is seen to consist of a number of lines of definite colours. The colours are found to be characteristic of the gases and so a calibrated spectroscope can be used to identify substances present in mixtures. When put into a non-luminous bunsen flame many solid substances are vaporised and give out their characteristic spectra. Spectrum analysis is a widely used method of identifying elements in their compounds.

The ***spectrum of sunlight*** and of the light from many of the stars is seen to be an almost continuous background crossed by a large number of thin black lines which indicate the absence of certain colours. Such spectra are produced when the white light from an intensely heated liquid or solid passes through gases at a lower temperature. The gases absorb more of their characteristic waves than they emit and so take out from the white light the colours which are characteristic of their own spectra.

Hence the positions of the black lines show which elements and compounds exist in the gaseous envelope of the sun.

The absorption of light by coloured transparent materials

If white light is examined by a spectroscope and then a piece of red glass is put in front of the slit, the spectrum is seen to lose nearly all the violet, blue, green and yellow.

It appears that the glass is red because it allows red light to pass through it but absorbs other colours.

Often, such a red glass is not a pure red but lets through some neighbouring colours of the spectrum.

Similarly a light blue glass may let through violet, blue and green while absorbing red, orange and yellow. Yellow glass often passes blue, green, yellow and orange while absorbing violet and red.

If you examine with a spectroscope the light *reflected* from red glass, you will find that it contains most of the colours of white light. This explains why it is that when red glass is powdered the powder often looks almost white since the light is now scattered from the many surfaces. (It is easier to try this experiment by powdering a piece of coloured sugar.)

The colours of opaque objects

A spectroscopic examination of coloured opaque surfaces shows that a red surface is red because it scatters more red light than light of the other colours which are falling on it. The scattered light is however usually much less pure than that which has passed through coloured glass.

The colour of a surface is thus dependent on the nature of the light which is falling on it. In a red light a red book is still red but a blue material usually appears a dirty black because it cannot scatter much of the red light which is falling on it.

Writing in red ink on white paper is almost invisible in the dark red light of a photographic dark room, but the writing appears black if viewed in a blue light. Since artificial light is rarely of the same spectroscopic quality as sunlight, materials which "match" in daylight may not appear to do so in artificial light and *vice versa*.

The colours produced by mixing paints

It is a well-known fact that a blue paint mixed with a

yellow paint produces a green mixture. The reason becomes evident if the spectra of the light scattered from the paints is examined. Blue paint scatters violet, blue and green, and absorbs most of the other colours. Yellow paint scatters green, yellow, and orange, and absorbs the blue light strongly. Hence the mixture can scatter strongly only the green when illuminated with white light. The colours of other mixtures can be explained on similar lines.

The colours produced when patches of light are superimposed on a white screen

A patch of pale blue light may on analysis show violet, blue and some green. Another patch of yellow may show some green, yellow, orange and red. If these two patches overlap on a white screen the screen will be receiving light of all colours and so may appear almost white.

In this sense then blue *light* and yellow *light* if mixed will produce white.

Two colours which when mixed give white are called **complementary colours.**

Invisible radiations

When a pure spectrum is produced on a screen it can be shown that, on either side of the rainbow band of colours, there are radiations which although they produce no visible effect on white paper or on the eye can yet be detected by other means.

If for example a screen painted with a little vaseline is put beyond the violet of the spectrum, it will be seen to glow with a curious " fluorescence " which shows that some type of radiation invisible to the human eye is falling on it.

A photographic plate exposed in this ultra-violet region will, after development, show very strong effects. The photographic emulsion is particularly sensitive to violet light and to the ultra violet radiations of still higher frequency.

At the other end of the spectrum, radiations of lower frequency than the red (" infra red " radiations) can be detected. These infra red waves were first detected by their heating effect when allowed to fall on the blackened bulb of an ordinary thermometer. Now they can easily be made to produce electrical effects on a thermo-electric

thermometer and photographic plates have also been made sensitive to them. The infra red rays are identical with heat waves and by using a camera and infra red plates it is possible in a dark room to take a photograph of a hot iron which is completely non-luminous to the human eye.

These invisible radiations have many of the properties of light. Thus they can be focused by using mirrors, preferably concave mirrors made of metal, and refracted through prisms and lenses of glass. Some types of glass are opaque to them although more or less transparent to ordinary light. You can now buy spectacles of glass made especially to cut off ultra violet radiation. Special window glass which absorbs a large proportion of the infra red is made for use in tropical countries.

Questions on Chapter XX

(**1**) What is meant by the dispersion of light ? On what fact does it depend ?

Show by means of a sketch how a spectrum is produced when a beam of white light passes through a prism. Explain what would happen if a piece of green glass were placed between the source of light and the prism. (O.)

(**2**) A parallel beam of white light passes through a converging lens and is then received on a white screen. In certain positions the light on the screen appears to be edged with red or orange and in other positions with blue. Give diagrams showing where the positions are and give reasons for these phenomena. (D.)

(**3**) If you had at your disposal a bright source of light, an adjustable slit, a prism and two lenses describe how you would arrange them to obtain the best possible spectrum on a screen ? Give reasons for the positions in which you would assemble the parts of the apparatus.

A photographic " dark room " should be illuminated with red or yellow light only. How would you test the coloured glass or fabric used in the dark room lamp in order to be satisfied that the light transmitted is safe ? (A.)

(**4**) Describe experiments with a prism to demonstrate the nature of white light. Explain how some objects when illuminated with white light appear to be coloured.

A picture is composed entirely of red and green paints. Describe how its appearance changes if it is illuminated, first with red and then with yellow light. (C.)

(**5**) Describe an experiment by which you could show that the different colours of which white light is composed have different refractive indices. For which colour is the refractive index greatest ?

A beam of red light is allowed to fall successively on a (*a*) black, (*b*) white, (*c*) red, (*d*) blue, surface. Describe and account for the various appearances presented. (N.)

LIGHT

RAPID REVISION TEST (1)

Observation shows that light travels in straight lines and does not bend appreciably as it passes opaque objects. Hence the shadows cast by objects can be drawn if we know the relative positions and sizes of the light source and the object. Diagrams (**1**), (**2**) and (**3**) show the shadows of a ball cast by a point source, a source smaller than the ball, and a source larger than the ball. The passage of light through a small hole produces pinhole images as illustrated in (**4**). Such an image consists of small patches of light each of which receives illumination from a particular point on the object. If for example the hole is triangular the patches are (**5**) and their size is given by equation (**6**). The amount of light passing from any point on the object to the screen is proportional to the (**7**) of the hole and so the exposure of a plate in a pinhole camera is necessarily long.

The illuminating power of a source of light is measured in units called (**8**). This unit is defined as (**9**). The intensity of illumination on a surface is measured in (**10**) and the relation between intensity of illumination and illuminating power for a small source is (**11**). Diagram (**12**) shows a (**13**) photometer. When this is used to compare the illuminating powers of two sources the screen of the photometer is set so that (**14**). Equation (**15**) gives the required ratio.

Light which falls on a surface may produce effects enumerated in (**16**). The two laws of reflection are (**17**) and (**18**). The paths of 3 rays from a point on the image of a point object placed near a plane mirror are illustrated in (**19**). A knowledge of the laws of reflection enables us to give a geometrical proof (**20**) of the position of this image.

Diagram (**21**) shows how the light from a point on the object passes to the eye of the observer who is looking at the image formed by a plane mirror. Figure (**22**) gives the appearance of the image of the letter M seen reflected in the mirror. A narrow beam of light reflected from the mirror

after falling on it at an angle of incidence A is shown in (**23**) which also shows what happens when the mirror is rotated through twenty degrees. Simple geometry summarised in (**24**) shows how the angle through which the reflected beam turns is related to the movement of the mirror. In two mirrors set at 60 degrees multiple images are formed as shown in (**25**).

The principal focus, the principal axis and the radius of curvature of spherical mirror surfaces are defined in (**26**). The geometrical proof that for mirrors of small aperture the principal focus lies half-way between the centre of curvature and the pole of the mirror is given (**27**). Geometrical diagrams illustrating the formation of images of an object placed successively at distances 4f, 2f, $1\frac{1}{2}$f and $\frac{1}{2}$f from a concave mirror are shown (**28**). For a convex mirror we get (**29**).

The algebraic relation between the distances of image and object from a spherical reflecting surface is (**30**). One convention adopted when using this formula is to call any distances (including the focal length) negative if they are (**31**). The magnification produced by the mirror is defined as (**32**) and it can be shown by geometry that this equals (**33**).

Refraction occurs when light passes from one transparent medium to another. The laws of refraction are (**34**). The physical explanation of the second law is that (**35**). Thus the refractive index of a substance really gives us the ratio (**36**). For glass and for water the values are approximately (**37**). Diagram (**38**) illustrates the effects when light falls on a glass-air surface at angles of incidence 20, 40, 60 degrees. The critical angle for two transparent substances is defined as (**39**). Its value for glass is calculated from the equation (**40**), and diagram (**41**) illustrates it.

A parallel beam of light passing through a rectangular block of glass is shown (**42**). For a triangular prism of glass the appearance will be (**43**), in which the angles of deviation and of dispersion are marked. If light is incident on a prism at various angles and the angle of deviation is plotted against the angle of incidence we get a graph resembling (**44**). From this graph an accurate value of the refractive index of the glass is obtained by using the formula (**45**).

Because of the refraction of light a fish in water is not seen in its actual position. When an eye is looking vertically downwards into the water the apparent depth is obtained by the relation (**46**). Diagram (**47**) shows how the apparent position of an object in water varies with the position of the observer's eye.

RAPID REVISION TEST (2)

LENSES AND OPTICAL INSTRUMENTS

Diagram (**1**) shows a convex and a concave thin lens with the positions of the optical centres and the principal foci marked. One convention is to call the focal length positive when (**2**) and negative when (**3**). The principal focus is defined as (**4**).

The focal length of a convex lens is found experimentally by (**5**). A very accurate value can be found by taking several values of the positions of an object and its real image and calculating the value of f from the formula (**6**). In this formula the convention about positive and negative distances is (**7**).

The focal length of a concave lens is found by (**8**) and using the formula (**9**).

The magnification produced by a lens is defined as (**10**). Its value is usually calculated from the equation (**11**).

Diagram (**12**) shows how three rays of light from the top of an object are brought together at a real image after passing through a convex lens as in a camera. The formation of a virtual magnified image is shown in (**13**).

The human eye is shown diagrammatically in (**14**). An eye can usually focus sharply on the retina the images of all objects whose distance from the eye is greater than about six inches, and the eye generally has a distance of most distinct vision of about ten inches. A short sighted eye has a lens which is too (**15**) and cannot see clearly objects which are (**16**). If the far point of such an eye be 40 inches the person should be provided with spectacles fitted with (**17**) in order that the lens shall form a virtual image of the distant objects at a distance of (**18**) from the eye. If the person reading with this lens holds a book 15 inches from his eye the image at which he looks will be at a distance of (**19**).

Long sight is due to the eye lens being (**20**) so that the person has difficulty in focusing objects which are near his eye. If his near point is 4 feet from his eye he will require spectacles with (**21**) lenses so that he can read in comfort. When he is holding a book 1 foot from his eye the glasses should give an image at (**22**). Hence their focal length should be (**23**). Long sight due to loss of the power of accommodation in old age usually requires two lenses, one to view distant objects and the other for reading.

A simple microscope or magnifying glass is formed by a (**24**) lens held usually close to the eye. Such a lens will give a magnification of 10 diameters if used with an eye whose distance of most distinct vision is 25 cm. when the object is at a distance of (**25**) from the optical centre of the lens. What is the focal length of this lens? (**26**).

A compound microscope is illustrated in diagram (**27**). Its magnifying power is defined as (**28**). This increases with decrease of the focal length both of the objective and the eyepiece.

Diagrams (**29**) and (**30**) show the optical construction and the images formed in the astronomical refracting telescope and the field telescope. Their magnifying power defined as in (28) can be shown to be equal to the ratio (**31**).

A pure spectrum is defined as one in which (**32**). It can be produced by the use of a slit, a convex lens and a glass prism arranged as in (**33**). The usual form of spectrometer is shown in diagram (**34**).

White light gives a (**35**) spectrum. The spectrum of sunlight differs from this because careful examination shows (**36**). Spectra of the light from incandescent gases consist of (**37**). Spectrum analysis enables us to discover the presence or absence of an element in a substance because (**38**).

White light becomes coloured after passing through blue glass because the blue glass (**39**). When white light falls on a blue glass surface a spectroscope shows that the scattered light (**40**). A mixture of blue and yellow paint appears green in white light because (**41**). If, by means of two lanterns, two patches of pale blue and pale yellow light are projected on a white screen the area in which they overlap appears (**42**) because (**43**).

SOUND

CHAPTER XXI

Sound Waves

Experience teaches us that bodies which are giving out loud sounds are in a state of vibration. This vibration can be felt if the finger is touched against the diaphragm of a loud speaker, the string of a violin or the throat of a singer. We know too that sound does not pass instantaneously through the air, but takes a definite time to travel. Thus we see a distant hammer fall an appreciable interval before the sound reaches us. In fact most of us have estimated the distance away of a lightning flash by counting the seconds between the flash and the thunder; $\frac{1}{5}$th of the number of seconds gives roughly the distance in miles. We know too that sound travels through other substances as well as through air. When we put our heads completely under water, we can hear the splash of water from a tap or a waterfall.

A simple experiment will show that sound, unlike light, will not pass through empty space. An electric bell is hung by rubber bands inside an airtight enclosure of thick glass which is joined to an efficient air pump. A small electric battery is connected to the bell to keep it ringing vigorously. The air is then pumped out of the enclosure and, when nearly all the air has been removed, it is noticed that the sound dies away although the hammer of the bell is still striking violently. If another gas such as hydrogen or carbon dioxide is now allowed to enter the enclosure, the sound of the bell is heard at once, showing that any other gas will carry the sound as well as air. Sound will in fact travel through solids, liquids or gases, but not through empty space.

The Measurement of Velocity of Sound in Air

The velocity of sound in air is easily determined by firing a gun and, from a known distance, observing, with an accurate stopwatch, the interval between the flash and the sound. If our stop watch reads to $\frac{1}{10}$ second and we want

to get the result accurate to within 1% we should want to have an interval of at least 10 seconds, that is the gun should be observed from a distance of at least 2 miles.

Two stations at an exactly known distance of about 5 miles are not difficult to arrange. If there is a gun and an observer at each station, it is possible to take the time interval from both stations and so to see whether any wind is affecting the result.

If the wind is blowing steadily and directly from one station to the other, the velocity of the sound in one direction will be velocity of sound in still air + velocity of air ; in the opposite direction it will be the difference between these velocities. Hence from the averages of many observations of the times in both directions, we could calculate both the velocity of the sound and of the wind. The temperature at the time of the experiment should be recorded because the velocity of sound is 1090 ft. per sec. at 0° C. but 1126 f.p.s. at 20° C.

Velocity of Sound in Water

A similar experiment can be used to find the velocity of sound in water. An explosion is made just below the surface of the sea (or of a fresh water lake) and the time recorded between the instant when the flash is seen and the arrival of the sound as received by a diaphragm placed in the water.

Echoes.

When a loud sound falls on a wall or a cliff we hear an echo of the sound and the interval between the direct sound and the echo depends on the positions of the source, the observer and the reflecting surface.

" Echo sounding " is now used to measure the depth of the sea beneath a ship. In one method a loud tap by an electrically controlled hammer sends out a sound which is reflected from the bottom of the sea. The time between the tap and the echo is recorded electrically and since the time is proportional to the depth, the scale of the instrument can be marked directly to read the depth ; the velocity of sound in sea water has to be determined from experiments where the depth is known.

Echoes and Reverberation in large rooms

When a loud sound is produced in a room, the sound is reflected in succession from the walls and is audible for some seconds until it gradually dies away owing to the losses of energy at each reflection. In a large empty hall with bare walls a sudden loud noise may continue to reverberate for as long as ten seconds. Such a hall would

be filled with a confused babble of sound if a speaker attempted to make a speech in it and his words would be unintelligible in most parts of the room. The presence of an audience may, however, so increase the absorbing power of the room that a loud sound will die away in little more than a second in which case the hall will be good for speech. In the same way a hall draped with heavy curtains and carpets or with specially prepared absorbent materials can be made free from excessive reverberation. In studios specially built for broadcasting great care is taken to obtain the best time of reverberation—the time differing slightly for speech and for various kinds of musical performances. The use of a sounding board behind a speaker is intended to direct towards his audience part of the sound which would otherwise escape upwards. Through a megaphone a voice carries further in a particular direction because the sound waves are less able to spread out in other directions. The megaphone also enables a speaker to set in vibration a greater volume of air and so he is able to give out more sound than he otherwise would produce.

The direction in which sound waves are reflected

It can be shown that when sound waves fall on a smooth hard surface they are reflected from the surface at an angle equal to that at which they fall on the surface. A simple but convincing experiment to show this is done by putting a watch inside a metal tube plugged at one end with cotton wool. This produces a " beam " of sound which is in a definite direction. Another metal tube is held against the ear. Then the ticking of the watch is only heard clearly when the two metal tubes are in line with each other or when they make equal angles with a reflecting surface. The law of reflection of sound waves can be stated as " The angle of reflection is equal to the angle of incidence."

The decrease in the loudness of sound with distance from the source

If sound were coming from a small source and spreading out uniformly in all directions, we should expect the sound energy reaching any small object such as the ear to vary inversely as the square of the distance.

This is because the area of the surface of a sphere is equal to 4π times the square of its radius. Hence when the sound waves had reached a distance r from the source they would be passing through an area of $4\pi r^2$.

Hence intensity of sound, if measured by energy passing per second through unit area at distances r_1 and r_2 would be as

$$S/4\pi r_1^2 \text{ to } S/4\pi r_2^2 \text{ i.e., } r_2^2/r_1^2$$

Actually the loudness of a sound as heard by a human ear does not vary directly with the intensity as defined by energy per unit area per second so that it is difficult to estimate how the loudness varies with the distance.

When sound passes down a smooth pipe (a speaking tube) it is found that if there are no sharp bends in the pipe the sound travels a very long way without appreciable loss of strength because it cannot spread out.

The nature of sound waves

The disturbances which, when they strike our ears, we call sounds, travel through the air or through any other material in the form of waves of increased and decreased pressure.

The waves or ripples on a water surface are called transverse waves because the displacement of the water is transverse to the line along which the wave is moving. Sound waves are not of this type, but are longitudinal waves, that is the motions of the air particles are backwards and forwards along the direction of the wave. For a loud note the actual motion might have an amplitude of $\frac{1}{1000}$th of an inch. Each particle moves with the same sort of motion as the bob of a long pendulum.

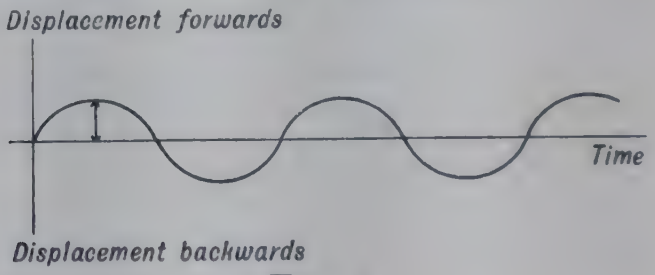


FIG. 1.

It is not easy to represent such a motion in a diagram except by a curve like that of fig. 1, which must be carefully distinguished from a transverse wave. It is intended to represent the *longitudinal* displacement of an air particle at successive intervals of time.

When a more complex sound passes, the motion of the particle may be very complicated, see fig. 2.

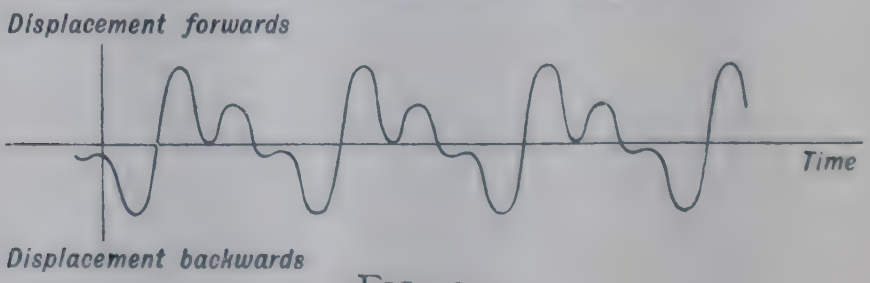


FIG. 2.

THE LOUDNESS of the sound at any instant is determined by the amplitude of the wave motion, i.e., the maximum displacement of the air particle from its undisturbed position.

THE PITCH of the sound depends on the frequency or number of vibrations occurring per second. High pitch sounds are heard when the number of vibrations is high, say over 1000 per second; low pitch sounds are produced by frequencies of about 100 per second.

THE MUSICAL QUALITY of a sound depends on the presence of more complex motions accompanying the vibration of lowest frequency. Many of these "overtones" can be heard by a trained ear in the note of a piano or other instrument (see p. 210).

Compressions and Rarefactions in Sound Waves

We will now consider the relative positions of neighbouring particles in the path of a beam of sound.

Let us represent the position of the undisturbed particles of some substance by a regular series of dots. (Fig. 3*a*.)

The particles move backwards and forwards as sound waves pass. Thus at some particular instant the positions of the particles may be as shown. (Fig. 3*b*.)

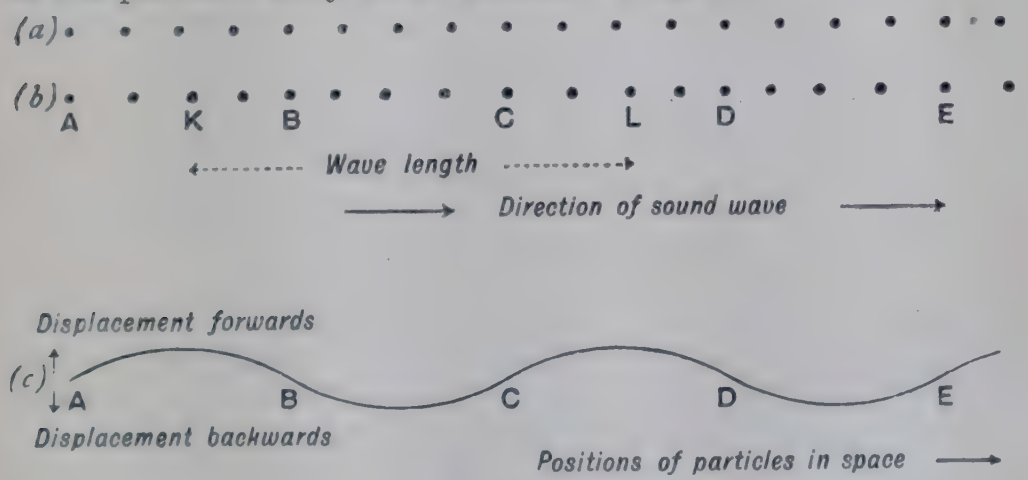


FIG. 3.

Some particles, between A and B, are displaced forwards; others, B—C, are displaced backwards and each successive particle is at a slightly different displacement relative to that of its neighbours.

The state of motion of a particle relative to its position of rest is called the *phase* of its motion and in a simple sound wave we find, at regular distances, particles in the same phase. Thus the particles at K and L are at their maximum displacement forwards. A, C and E are in the middle of portions of material where the particles are further apart than usual—that is at the centre of a rarefaction. B and D are the centres of successive compressions, since near them the particles are nearer together. The distance KL, AC, or BD is called a wave length.

Diagrams 3 (*a*) and (*b*) are difficult to draw quickly and so the various parts of the sound wave are usually represented as in fig. 3 (*c*). Again we must be careful to distinguish this graph from a diagram of a transverse wave or from fig. 1 which we said represented the displacements of one particle at successive instants of time. The only safe way to avoid confusion with these diagrams which look so much alike in shape is to be very careful to mark along the axes exactly what the curves are intended to represent. We should note that in fig 3 (*c*) the points A, C and E represent rarefactions and B and D show compressions.

The relation between velocity, frequency and wave length

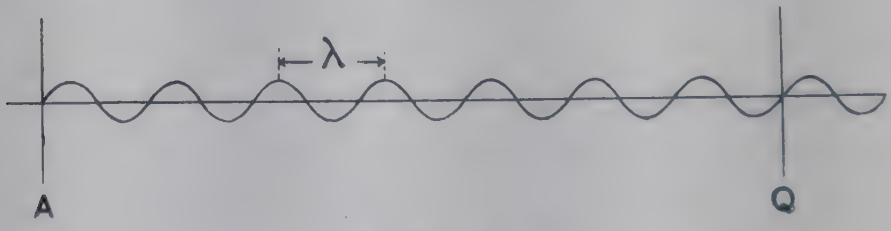


FIG. 4.

There is a simple relation between the velocity (V), frequency (n) and wave length (λ) of a train of waves. Suppose in fig. 4, that n waves pass a point A in one second. Then if V is their velocity, the wave which was passing A at the beginning of the second will have reached Q where AQ = V. Hence n waves must lie between A and Q so that

$$V = n.\lambda$$

It is well to remember that the frequency of a set of waves remains unaltered when the waves pass from one

substance to another but that the velocity changes, and so the wave length is changed in the same proportion.

The experimental measurement of frequency of vibration

The vibrations of most sounding bodies occur much too rapidly for their frequency to be determined by any direct counting of the vibrations in a measured time. It is usual therefore to determine the frequency by reference to the known frequency of a tuning fork which has been bought as a standard of musical pitch.

To measure the frequency of a tuning fork which is unmarked or to verify that of one which is marked, the simplest method is to fix to a prong of the fork a tiny bristle with a very small piece of wax or gum and let the vibrating bristle " write " its trace on a smoky surface moving at a known speed.

Experiment to measure the frequency of vibration of a tuning fork

Take the fork and sound it with another fork of the same pitch. If they are exactly in tune they will produce a smooth sound ; if one is very slightly out of tune with the other, you will hear variations of loudness of the sound called beats (see p. 218).

Now fix the bristle to project about half an inch from the end of a prong of the fork and again sound the two forks. If they are still exactly in tune or produce exactly the same number of beats per second, you will know that the attachment of the bristle has not altered appreciably the frequency of vibration.

Place on a gramophone turntable a disc of wood and round the edge of this disc fix a long thin strip of paper which has been blackened by passing it rapidly through a smoky flame. Let the disc rotate and time 100 revolutions with a stop watch. If l cm. is the length of the circumference of the disc and the speed of rotation is n turns in t seconds, the velocity of a point on the paper strip will be $\frac{nl}{t}$ cm. per second. While the disc is rotating uniformly, strike

the fork and then hold it so that, for an instant, the bristle touches the moving paper with the vibrations of the bristle occurring at right angles to the direction of motion of the paper. Do this two or three times. On the paper should be found several series of waves like fig. 4. Measure carefully the total length d of a number x of the waves and so find accurately the distance $\lambda = \frac{d}{x}$ the paper has travelled during one complete vibration of the fork. Then the number of vibrations of the fork in one second is the velocity of the paper divided by this length.

The dropping plate method for finding the pitch of a tuning fork

A rather more complicated method is to let the bristle write a trace on a smoky plate of metal or glass which is falling freely in a vertical plane. The plate is hung in the required position by a thread so that when the thread is burnt, the plate will drop smoothly from rest with the acceleration " due to gravity " of a freely falling body.

The wave trace obtained shows a wavy line which marks how far the plate has fallen for each vibration of the fork, the earlier marks being difficult to see clearly. Suppose that we measure 20 clearly marked vibrations and find that these occurred between distances l_1 and l_2 measured from the position of the plate before it began to fall (fig. 5). Then if t_1 is the time taken to fall the distance l_1 and t_2 is the time for l_2 we have from the equations for the motion of a falling body

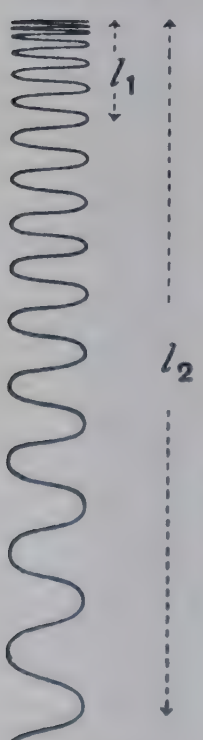


FIG. 5.

$$l_1 = \tfrac{1}{2}gt_1^2 \quad \text{i.e.} \quad t_1 = \sqrt{\frac{2l_1}{g}}$$

$$l_2 = \tfrac{1}{2}gt_2^2 \qquad t_2 = \sqrt{\frac{2l_2}{g}}$$

where g is the acceleration due to gravity. Also $t_2 - t_1$ is the time of twenty vibrations so that we can find how many vibrations occurred in 1 second since

$$\text{frequency (n)} = \frac{1}{\text{time of one vibration}} = \frac{1}{(t_2 - t_1)/20}$$

$$= \frac{20}{\sqrt{\frac{2l_2}{g}} - \sqrt{\frac{2l_1}{g}}}$$

Example.

The plate dropped from rest through a distance of 5 cm. and, while it was dropping through the next 15 cm., 25 vibrations were made by the fork. Here $l_2 = 15 + 5$ $l_1 = 5$

$$\text{Hence frequency} = \frac{25}{\sqrt{\frac{40}{981}} - \sqrt{\frac{10}{981}}} = 247{\cdot}6$$

The Siren

The simplest form of siren is a disc of metal or cardboard with sets of equidistant holes punched in it along the circumference of one or more circles. A blast of air from a small jet connected to bellows or to a compression air pump is set so that it will blow through a set of holes when the disc is rotated. The disc is usually mounted on the shaft of an electric motor of which the speed can be regulated.

When the disc is rotating the succession of puffs of air sent through the holes produces a note of which the frequency can be calculated as $n = N.s$ where N is the number of holes in use on the disc and s is the speed in revolutions per *second*.

The siren can be used to measure the frequency of any note by speeding up the disc until the note of the siren is exactly in tune with the unknown note. The speed of the disc is then kept constant for, say, one minute and the number of revolutions of the disc measured by an automatic recorder.

The siren disc is often punched with several rings of holes of which the numbers are in the ratios of 24 : 30 : 36 : 48. If the air jet is allowed to play on each set of holes in succession the notes of the common chord are heard whatever the speed of the disc. Thus the musical interval which we recognize as an " octave " must depend only on the ratio of frequencies being 1 : 2 and not on the actual frequency of any note.

Experiments on Strings

(1) Experiments show that the frequency n of vibration of a stretched string varies with changes in its length l, its tension T and

the mass per unit length m of the string, and is given by the formula

$$n = \frac{k}{l}\sqrt{\frac{T}{m}}$$

where k is a constant depending on the units in which the various measurements are made.

To illustrate this law a single string or wire (a sonometer) is kept under tension by weights hanging over a pulley wheel. The length of the string can be altered by moving a " bridge " on which one end of the vibrating part of the string rests, and its tension is measured by the weights hung over the wheel. A number of tuning forks with their frequencies marked on them are used, the sonometer being tuned to the various forks.

(2) The form of a vibrating string.

If the appearance of a vibrating string is carefully noticed it will be seen that many different types of motion are possible and a musically trained ear will recognise that different overtones are present in the resulting sound. These differencies of quality are most easily produced by plucking the string at different places along its length.

The particular overtones present can be made obvious to all listeners by stopping some of the vibrations of the string by touching it at various points with the edge of a handkerchief.

You should pluck the string near one end and then try the effect of touching it with a handkerchief at distances of $l/2$, $l/3$, $l/4$, $l/5$, where l is the total length of the string.

You will hear in succession the various harmonics of the tone of the lowest pitch—the octave, twelfth, and double octave, etc. Careful observation shows that when the string is sounding these harmonics clearly it takes the forms illustrated in fig. 6.

Points on the string which are in most violent motion are called *antinodes* or loops while the points of practically no apparent motion are *nodes*.

FIG. 6.

Stationary Waves and Progressive Waves

When a string is vibrating in any of the forms of fig. 6 its form illustrates " stationary waves " that is, waves which keep a fixed position in space because there are points of no motion (nodes) which do not change their position. It is often convenient to think of such waves as if they were produced by the simultaneous passage of two sets of progressive waves of equal wave length travelling in opposite directions. Stationary waves are also produced in resonant tubes and, by reflection of sound, in front of smooth surfaces.

Harmonics and Overtones.

The notes of frequencies 2n, 3n, 4n, etc., are called the harmonics of the fundamental note of frequency n.

When a musical instrument is used to produce a sound, a trained ear can usually recognise other notes besides that of the lowest frequency which is called the fundamental. These other notes are called overtones. They may or may not be harmonics of the fundamental. With an organ pipe most of the overtones are harmonics but with a bell several inharmonic overtones are produced.

Questions on Chapter XXI

(**1**) Describe and explain experiments to show that—

(*a*) sound does not travel across a vacuum ;

(*b*) the velocities of sound in air and in water are different. (O.)

(**2**) On what factors does the velocity of sound in the atmospheric air depend?

Describe, giving all necessary details, how you would by a laboratory experiment determine the velocity of sound in air. Indicate any corrections which you may require to make in your readings. (N.)

(**3**) How are echoes produced ? Make a careful diagram illustrating how the waves from a source of sound are reflected at a plane surface.

An observer at a certain distance from a cliff notes that the interval between a sound he makes and its echo is 3 sec. He then walks 550 ft. nearer to the cliff and finds that the corresponding interval is 2 sec. Calculate (*a*) the velocity of sound, (*b*) the observer's original distance from the cliff. (N.)

(**4**) A man standing between two parallel cliffs fires a rifle. He hears one echo after $1\frac{1}{2}$ sec., one after $2\frac{1}{2}$ sec., and one after 4 sec. Explain how these echoes reach him, and calculate the distance apart of the two cliffs. The velocity of sound is 1120 feet per second. (L.)

(**5**) Explain the following :—

(*a*) Broadcasting studios are sometimes hung round with curtains of some heavy soft material.

(*b*) Pulpits in churches are sometimes provided with overhead sounding boards.

(*c*) A megaphone is used to enable a speaker's voice to be heard at a distance. (N.)

(**6**) How would you show that sound waves are reflected by a hard surface in accordance with a definite law? State this law.

Explain why the sound of your voice appears to you to be much louder when you are speaking in an empty hall than when you are speaking in the same hall filled with people. (N.)

(**7**) Explain, with the aid of a diagram, what is meant by longitudinal wave motion. What characteristics of the wave determine (*a*) the pitch, (*b*) the loudness, (*c*) the quality of a musical sound? (L.)

(**8**) Sound waves are passing through a substance. Describe with the help of careful diagrams (*a*) the movement of a particle of the substance, (*b*) how this movement changes from particle to particle along the line of propagation.

Describe briefly one method of measuring the velocity of sound in air. (C.)

(**9**) Explain what you understand by the *velocity*, *wavelength*, and *frequency* of a wave motion, and deduce the relation between them.

A heavy car, with front and rear wheels an integral number of feet apart, runs over a road made of round logs 1 ft. in diameter laid transversely, and touching one another. What must be the velocity of the vehicle if the jarring of the wheels on the road produces a note of wavelength $12\frac{1}{2}$ ft.?

(Velocity of sound = 1100 ft. per sec.) (O.)

(**10**) What is meant by the statement that the frequency of a tuning fork is 256? Give any *one* experimental method for determining the frequency of a given tuning fork. (N.)

(**11**) Describe briefly the nature of the disturbance in air which gives rise to a musical note. What is the essential difference between the disturbances which reach the ear when a note and its octave are heard?

Describe any method by which you could determine the frequency of a note sounded by a distant source. (N.)

(**12**) Explain what is meant by (*a*) *frequency*, (*b*) *wavelength* of a sound in air. Deduce the relation between these quantities and the velocity of the sound.

Describe a siren and explain how it can be used (*a*) to determine the frequency of the note of a given tuning fork, (*b*) to investigate the connection between a note and its octave. (N.)

(**13**) Describe a simple type of siren and explain how you would use it to show (1) that the pitch of a note depends only on the frequency of vibration of the source, (2) that the difference in pitch of two notes or the musical interval between them depends only on the ratio of their frequencies. (L.)

(**14**) What is meant by the pitch of a musical note? On what factors does the note emitted by a stretched wire, when plucked or bowed, depend?

Two stretched wires A and B, of the same material and same diameter, give, when plucked, notes whose frequencies are in the ratio 1 to 2 respectively. If the length of A is twice that of B, compare the tensions in A and B. (N.)

(**15**) Explain briefly how you would show by experiment the relation between the length and the frequency of a vibrating string which is stretched by a constant force.

A string is stretched by a constant force and is found to emit a note of frequency 250 vibs./secs.; when the length of the string is reduced by 30 cm. the frequency of the note given out is now found to be 400 vibs./sec. What was the original length of the string? (L.)

(**16**) You are required to find the relation between the tension in a stretched string and the frequency of the note which it emits when plucked or bowed. Explain in detail how you would proceed.

In such an experiment the following results were obtained :—

Frequency,	260,	320,	380,	480,	520.
Tension,	1·69,	2·56,	3·61,	5·76,	6·76 lb. wt.

Explain and show how these results may be used to plot a graph to show in the clearest manner possible the relation between the frequency and the tension. State in words the conclusion to be drawn from the results.

Give a diagram of a string vibrating in four equal loops, show on it (*a*) a node, (*b*) an antinode, and use the diagram to explain the meaning of the terms (i) harmonic, and (ii) overtone. (O.)

(**17**) What is a *node* and an *antinode*?

Describe an experiment to show their production on a stretched string, indicating where they would be expected.

A stretched string is made to vibrate with (*a*) its fundamental note, (*b*) its first overtone. If the frequency of the fundamental note is 300, calculate the wave-length of the note and of its first overtone.

(Velocity of sound = 33,150 cm. per sec.) (O.)

(**18**) Explain carefully the terms *longitudinal, transverse, progressive, stationary,* as applied to wave motion. Illustrate each of the above terms by reference to the following example.

A string of length 50 cm. is plucked and a note of frequency 250 is heard. Find the wave length of the vibration (*a*) in the string, (*b*) in the air.

[Velocity of sound in air = 340 metres per sec.] (N.)

CHAPTER XXII

Experiments on Columns of Air. Resonance.

If a tube of glass or metal is held vertically with one end below the surface of water (fig. 7), it will be noticed that any particular length of air column AB has a characteristic frequency of vibration which can be made audible by blowing across the top of the tube, or by tapping the tube with a pencil.

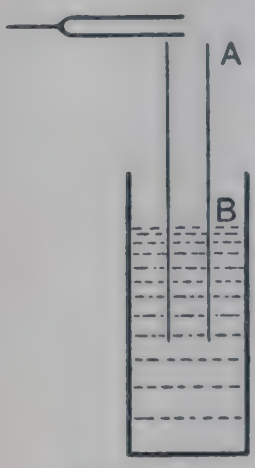


FIG. 7.

If now a tuning fork is sounded and held just above the open end of the tube, it will be found that very little effect is produced until the natural note of the air column AB is adjusted to be as nearly as possible the same as that of the tuning fork. At this particular length the air column is set in violent oscillation and a loud note sounds.

We say that there is resonance between the sounding fork and the air column. By measuring the length of tube which will resonate to a fork of known frequency, we can calculate the velocity of sound in the air of the tube (see below).

The essential condition for resonance

If a heavy object is hung by a rope, it will have a natural period of vibration. Under what conditions can large oscillations of the body be produced by small forces? Experience shows that the small pushes must be in time with the natural period of the swinging body and in the direction in which it is moving. With a push of definite strength the maximum effect is obtained by pushing every time the body is moving away; a resonance of less amplitude is produced by pushing every other time; a push every third time gives a still less vigorous resonance.

The necessary mechanical condition is then that the driving forces shall act with the same frequency as that of the oscillator or with frequencies exactly $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of its natural period.

The motion of the air particles in a resonating air column

The motion in the tube AB (fig. 7) can be shown to be of the type known as a "stationary wave." The air particles near A are vibrating violently with amplitudes of perhaps $\frac{1}{100}$ cm. Further down the tube the amplitude of vibration is less until at B the amplitude is very small indeed.

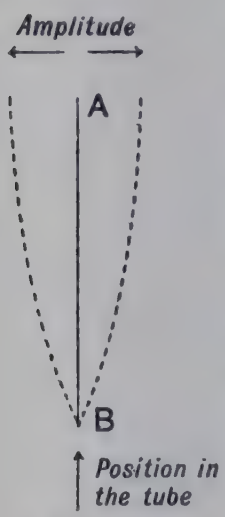


FIG. 8.

The point B where there is no motion is called a *node* and A is a point of violent motion, an *antinode* or loop (see fig. 8).

If we now compare such a stationary wave with the sound wave travelling through air (fig. 3), we notice that the wave length of the sound wave was four times the distance from a point (C) where the air was undisturbed to the point (L), where a particle was experiencing its maximum displacement.

Similarly the length of the stationary vibration in a resonating air column closed at one end is $\frac{1}{4}$ of the length of the sound wave.

Hence if l is the length of the tube, the wave length of the sound in air is 4l and so the velocity and frequency are connected by the relation

$$V = n\lambda = n.4l$$

This is the correct formula for a tube of small diameter, but experiment shows that for a wide cylindrical tube the exact formula is $V = n4\ (l + \cdot 3d)$ where d is the diameter of the tube. $(l + \cdot 3d)$ is often called the "corrected" length of the tube and $\cdot 3d$ the "open-end" correction.

A tube rather more than 3 times as long as the above tube will be found to give a marked resonance with the same tuning fork. The stationary wave set up in this case is illustrated in fig. 9.

Experiments with the resonance tube are often used either to calculate V if n is given or to find the frequency n of an unknown fork assuming the value of V to be known.

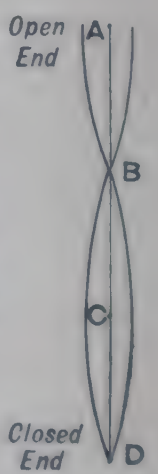


FIG. 9.

The resonance of a tube open at both ends

When a tube open at both ends is sounding its natural note the air particles at the ends are in violent motion and at the middle there is a node. (Fig. 11.)

Here the length AB is approximately half a wavelength and the formula connecting the velocity of sound, the frequency and the length of the tube is

$$V = 2n\,(l + \cdot 6d)$$

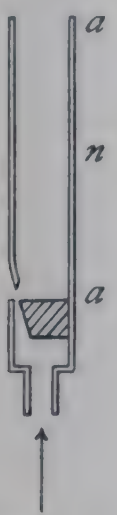


FIG. 10.

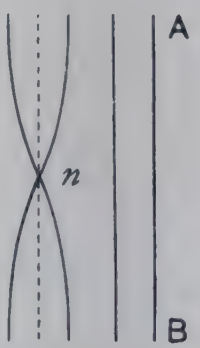


FIG. 11.

The organ pipe

The organ pipe consists of a tube in which vibrations are produced when air is blown into the pipe. Fig. 10.

An open organ pipe of simple design has the tube open at both ends. At the bottom air from the bellows is directed through a thin jet on to a thin lip. Oscillations are set up there and the air rushes in and out at the lip and sets up an oscillation of the type illustrated in fig. 10, with a node at the centre and antinodes at the two open ends.

The frequency of oscillation is thus given by

$$V = 2n\,(l + c)$$

where c depends on the end corrections which vary with the size and shape of the openings.

Overtones of an open pipe

If the pipe is blown with a steadily increasing pressure the quality of the sound changes and suddenly the pipe

sounds its first overtone—the octave of frequency twice as great (2n). The oscillation can then be shown to have two nodes (fig. 12*a*). Still greater pressure of blowing gives in succession the 2nd, 3rd, etc., overtones of frequencies 3n, 4n (figs. 12 *b*, *c*).

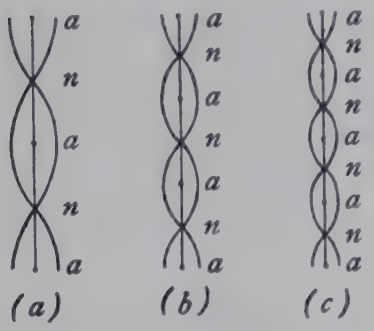


FIG. 12.

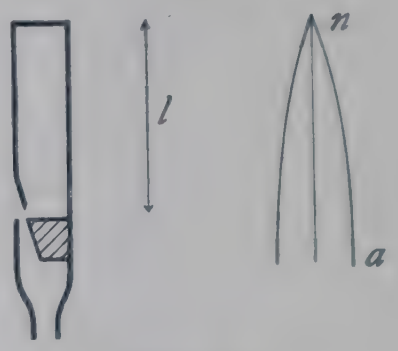


FIG. 13.

The open type of oscillation is given also by the whistle pipe, flute, and piccolo.

The closed organ pipe, fig. 13, gives an oscillation which has a node at the closed end and an antinode at the open lip. Hence its frequency is given by

$$V = 4n\ (l + c')$$

where c′ is the end correction of the instrument. Thus it produces the same fundamental note as an open pipe of twice its effective length.

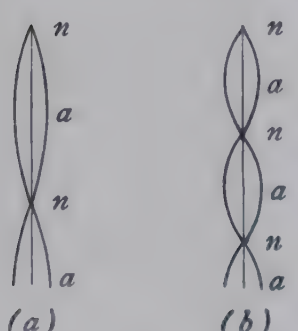


FIG. 14.

Its overtones, produced by "overblowing" it, are however of frequencies 3n, 5n, 7n, etc., since the air oscillations in it have always a node at the closed end and an antinode at the open lip. Hence the types of oscillation are as shown in figs. 14 (*a*) and (*b*).

Beats

If two organ pipes of nearly the same length are sounded, the notes produced are of nearly the same pitch. When the two are sounding simultaneously the loudness of the resulting tone fluctuates regularly, producing "beats."

It can be shown that the number of beats heard per second equals the difference of the frequencies of the two

notes. Also when the frequencies of the beats get too high to be counted easily a sense of "discord" is produced. This becomes very pronounced when the beat frequency is about 30 per second.

Similarly if two tuning forks of nearly the same pitch give beats, say n per sec. and if the frequency of one of the forks is known (N), that of the other will be either $N + n$ or $N - n$. Load the unknown fork with a small piece of plasticine or wax so that it will vibrate more slowly. Again count the beats between it and the standard fork. If they have increased, the unknown fork must have a frequency of $N - n$.

The effect of temperature on the frequency of an organ pipe

Since the frequency of a closed pipe is given by $n = \frac{V}{4l}$ and that of an open pipe by $\frac{V}{2l}$, the actual frequency varies with the velocity of sound in the gas (normally air) used to sound the pipe.

It can be shown that the velocity of sound in a gas at any temperature t° C. is related to its velocity at 0° C. by the formula

$$V_t = V_0 \sqrt{\frac{273 + t}{273}}$$

Also since V_0 is very nearly 1090 ft. per sec. or 33,200 cm. per sec. the above equation is approximately the same as

$$V_t = \begin{cases} 1090 + 2t \text{ feet per second.} \\ 33{,}200 + 60t \text{ cm. per second} \end{cases}$$

Hence the rise in frequency when the temperature rises can easily be calculated from this last equation, for air.

The change of pitch heard when a sounding body passes an observer. (The Doppler Effect.)

At the moment when a sounding body such as the whistle of a locomotive passes an observer, the pitch of the sound drops suddenly by an amount which is about a semitone for a body moving at 50 miles an hour past an observer at rest.

Doppler first explained the causes of this effect which is therefore known by his name.

A sound wave given out from a moving body spreads out as a sphere round the particular positions of the body at the instant the wave is being emitted.

Hence in front of the moving body there are more waves in a given distance than there would be if the body had not been moving. The wave length in front is therefore shortened while behind the wave length is lengthened. Hence an observer towards whom the source is moving receives more waves per second and hears a note of higher pitch than that actually emitted from the source. An ear from which the source is receding hears a note of lower pitch.

Also if the observer is moving towards the source he will be affected by more waves per second than would pass another observer who is at rest. Hence a motion of the observer towards the source of sound causes a rise in the frequency of the note heard and motion away from the source produces a fall in the pitch of the sound heard.

It is easy to work out a formula for the frequency which will be observed under any conditions.

Let velocity of sound in air = V

Let velocity of the source from A towards B be V_s (fig. 15).

Let velocity of the observer from C towards D be V_o.

FIG. 15.

Then if AB = distance the source travels in one second, the wave which was emitted from A will have gone a distance V by the time the sound is at B. Hence wave length of the sound in front of B is

$$\frac{V - V_s}{n}$$

Suppose that at a certain instant the observer is at C and that after one second he has moved to D.

The waves which have passed him during the second will now be spread over a distance $V - V_o$

Hence the number of waves $\left(\text{of wave length } \frac{V - V_s}{n}\right)$ which have passed him must be $\frac{V - V_o}{\lambda} = \left(\frac{V - V_o}{V - V_s}\right)n$

This expression is easily simplified for cases in which

$$V_o = 0 \text{ or } V_s = 0.$$

Questions on Chapter XXII

(1) Describe an experiment to illustrate the phenomenon known as resonance.

Explain, with a diagram of the apparatus, how the velocity of sound in air may be determined by the use of a

tuning-fork of known frequency and a vibrating column of air. (O.)

(**2**) If a test tube is partially filled with water and then struck with a pencil a musical note is heard whose character depends on the dimensions of the tube and on the quantity of water. Explain this, saying how the note varies with these factors.

If a half-inch tube gives a note of frequency 1280 when filled to within 2·25 in. of the top, what is the velocity of sound in air ? (N.)

(**3**) If the velocity of sound in air is 1120 ft. per second, what is the wave-length in air of the note emitted by a tuning fork of frequency 384 ? How would you determine this wave-length experimentally ? (L.)

(**4**) Why does a vibrating tuning fork sound louder when it is mounted on a hollow wooden box than when it is held in the hand by its stem ?

What should be the approximate length of a box, open at one end, which should be used for a fork of vibration frequency 256 to ensure maximum loudness ?

(Velocity of sound in air may be taken as 1100 ft. per sec.) (N.)

(**5**) Explain what is meant by resonance and describe two experiments in illustration.

Describe the motion of the air in an open organ pipe when it is sounding its fundamental note. (C.)

(**6**) Describe briefly the construction of an organ pipe. A given organ pipe, open at both ends, sounds a certain note. What will be the effect on the pitch of the note if the top of the pipe be closed, and why ?

Explain why the " bass " pipes of an organ are long but the " treble " pipes short. (N.)

(**7**) Describe how a note is produced by a siren and explain how, with its aid, you would determine the frequency of the note of an organ pipe.

What would you expect to be the frequency of the note of an open organ pipe two feet long, on a day when the velocity of sound in air is 1120 feet per second ? (C.)

(**8**) How would you determine the frequency of vibration of a tuning fork?

Two tuning forks have nearly the same pitch. Describe and explain how the difference of their frequencies can be found. (C.)

(**9**) When two tuning forks, A and B, are sounded together, the intensity of the sound is found to increase and decrease periodically. Explain this.

It is found that the sound dies away completely twice every second. If a little wax is put on the prong of fork B, the variations of intensity are found to occur more rapidly. If the frequency of fork A is 256 per sec., what is the frequency of fork B? (C.)

(**10**) How is the velocity of sound in air affected by change of temperature?

Explain the change in pitch of wind instruments as the air temperature varies. (L.)

Sound

Rapid Revision Test

Sound travels through air with velocity (**1**) but will not pass through (**2**). Sound waves consist of (**3**) and the motion of particles of the substance through which the waves are passing can be represented by the figure (**4**).

The loudness, pitch and quality of a sound are determined by (**5**). The relation between velocity, wavelength and frequency is (**6**), and when waves pass into another medium the (**7**) remains unchanged.

The frequency of vibration of a tuning fork can be accurately measured by the experiment (**8**) while that of an organ pipe is determined by (**9**).

A note and its octave are related by (**10**).

The frequency of vibration of a string can be calculated from the formula (**11**). This can be shown experimentally to be correct by using the apparatus shown (**12**). A stretched string can sound its overtones if it is made to vibrate as in figure (**13**). Waves on a string are called (**14**). The points of most violent motion are called (**15**) and the points at rest are (**16**). Resonance occurs when (**17**). When the air in a glass tube is resonating to a vibrating tuning fork the motion of the air particles is as shown in (**18**).

Hence the frequency of vibration is related to the length of the tube by the equation (**19**). With a cylindrical tube closed at one end the corrected length is given by (**20**).

When an open organ pipe is blown too vigorously it sounds its overtones and their frequencies are (**21**). A closed pipe has the overtones (**22**).

Beats are produced when (**23**). The number of beats per second gives (**24**).

Rise of temperature alters the frequency of an organ pipe because (**25**). When a source of sound is moving towards an observer the pitch of the note heard is (**26**). This effect was explained by Doppler whose explanation was (**27**).

ANSWERS

CHAPTER I

3. 44·8 lb., 4·48 gall. **4.** 2·6. **5.** 2·5, 33·6%. **7.** 1·29 gm. per litre. **9.** (*a*) 1 : 2·62, (*b*) 1 : 4·71. **10.** 0·553 gm. **11.** 0·91. **12.** 2·17%.

CHAPTER II

1. (*a*) 64·06 lb. per sq. ft., (*b*) 64·06 lb.; 2062·5 lb. per sq. ft., 4125 lb. **2.** 1173 lb. per sq. in. **3.** $1{\cdot}11 \times 10^7$ dynes per sq. cm. **4.** 10 gm. per sq. cm.; 3142 gm. wt., 2094 gm. wt. **6.** 0·145. **7.** 7 cm. **8.** 34 ft., 42·5 ft. **10.** 1020, 1020 cm. **11.** 667 mm. **12.** 2·97 tons. **14.** 78 cm. **15.** 18 lb. per sq. in. **16.** 22·1 cm. **17.** 680 litres. **18.** 0·9 cu. in. **19.** 76 cm. **20.** 6 cu.m. **21.** 225·8 cm. **22.** 450 cu. ft. (if originally full of water). **24.** 20·67 kgm. **25.** 4·64 in. **26.** (*a*) 85⅓. (*b*) 94·8. **28.** 2020. **29.** 59. **30.** 50 cm.

CHAPTER III

1. 127·8 gm. **2.** 8·9, 18·854. **3.** 2⅔; 0.8. **5.** 108 gm., 104 gm., 7 cm. **6.** 50⅝ lb.; 6⅙ lb. **8.** 19·45 gm. **9.** 8·9. **11.** 3 c.c. **12.** 11,100 kgm. **13.** 2770 lb. **14.** 15 c.c. **15.** 1·28 in. from bottom; 21,700 lb. **16.** 4 kgm. **17.** No. **18.** 23·8 c.c. **19.** 1·08. **20.** 12·5 cm.; 2 cm. **21.** 1·05 gm. per c.c. **22.** (*a*) 0·84 gm. per c.c., (*b*) ⅔ gm. per c.c.; 64 gm. **23.** 62 gm.; 0·9. **24.** 1·00 — 1·43. **25.** 18 cm. **26.** 6·3 cm. **27.** 2·70 gm. per c.c. **28.** 13·14 ft.

CHAPTER IV

1. 15·9° C., 60·7° F. **3.** 22·8 centigrade degrees, 17·2° C. **4.** — 40°. **5.** —6·7° C., 48·9° C. **6.** 35° C., 43·3° C. **7.** 25° C.

CHAPTER V

1. 0·35 in. total expansion. **2.** 0·000010. **3.** 266° C. **4.** 0·000011. **5.** 800° C. **6.** 0·0051 sq. cm. **7.** 51·023 cm.

CHAPTER VI

1. 0·000514; 0·000541. **2.** 0·000443. **3.** 0·0357 cm. **4.** 3⅓ c.c. **5.** 15·8 c.c.

CHAPTER VII

1. 23·3° C. **2.** 2·92 atmos. **3.** 21·3. **4.** (*a*) 70 cm. (*b*) 273° C. **5.** 59·5° C. **6.** 1·207 gm. **7.** 0·47 gm.

CHAPTER VIII

1. 0·113. **2.** 1·3d. **3.** 40° C. **4.** 21,750 cals. **5.** (*a*) 5·63 cals., (*b*) 5·63 gm. **6.** 1215° C. **7.** 0·25. **9.** 0·51. **10.** —181° C. **11.** 0·54. **12.** 0·65; 9600 cals. **13.** 7195 cals. **14.** 2000 gm. **16.** 35′ 35″. **17.** $\frac{1}{18}$; $9\frac{1}{2}$ min. **18.** 21·5 gm. **19.** 50·3 cals. per gm. **20.** 86·6%.

CHAPTER IX

1. 8·95 gm. **3.** 528 cals. per gm. **6.** (*a*) 1000 gm., (*b*) 593,000 cals.

CHAPTER X

1. 2000 cals. per sec. **2.** 0·3 cm. **8.** 0·73.

CHAPTER XI

1. 0·23° C. **2.** 1414 cm. **3.** 257 cals. per gm. **4.** $4{\cdot}42 \times 10^7$ ergs per cal. **5.** 49 ft. lb. per rev. **6.** (*a*) 364 cals., (*b*) 1130 ft. lb., (*c*) 3·1 ft. lb. per cal.

CHAPTER XII

4. 120 ft.

CHAPTER XIII

1. 3·87 ft. **2.** 90 c.p.; 17·6 c.p. **3.** 57 c.p. **4.** 2·24 ft. **5.** (*a*) 20 cm., (*b*) 60 cm., from 6 c.p. lamp. **6.** 152 : 125 : 27. **8.** 0·6 ft. from former.

CHAPTER XV

2. 2 ft. 3 in., 1 ft. $1\frac{1}{2}$ in. **3.** 2 ft.; 7 ft. **6.** 45°. **7.** 60°. **9.** 6 (2 coincident).

CHAPTER XVI

2. 42 cm.; 3/7. **3.** (*a*) 4 in., (*b*) 2 in. (**4**) (*a*) 60 cm. behind mirror, 16 cm. long, erect, virtual, (*b*) 31·4 cm. in front of mirror, 2·3 cm. long, inverted, real. **5.** 240 cm.; 48 cm. **7.** $3\frac{1}{2}$ cm. behind mirror, $\frac{2}{3}$ cm. erect. **8.** Image $\frac{1}{4}$ in. long; its nearer end $3\frac{1}{2}$ in. behind mirror.

CHAPTER XVII

2. 2·04 in. **5.** 0·26 cm. **13.** (*a*) at centre, (*b*) $3\frac{9}{16}$ in. from near side.

CHAPTER XVIII

1. 16 in. away. **2.** 1·02 in. **3.** (*a*) 10 in., (*b*) 9·6 in. **5.** 25 cm. behind lens, real, inverted, 1 cm.; virtual, erect. **6.** 8·08 in. **7** (*a*) $9\frac{1}{3}$ in. in front of lens, $4\frac{2}{3}$ in. high, virtual, erect; (*b*) $23\frac{1}{3}$ in. behind lens, $4\frac{2}{3}$ in. high, real, inverted. **10.** (*a*) 20 cm., (*b*) $6\frac{2}{3}$ cm. in front of lens. **11.** Convex, 15 cm.

CHAPTER XIX

1. 8 in., 2·4 in. **2.** 11 ft.; 7·56 sq. ft. **3.** The 20 in. **4.** Concave, 30 in. **6.** Convex, $33\frac{1}{3}$ cm. **7.** 15 in. **9.** $4\frac{1}{6}$ cm. from object; 6. **10.** 6 in.

CHAPTER XXI

3. 1100 ft. per sec.; 1650 ft. **4.** 2240 ft. **9.** 88 ft. per sec. **14.** 1 : 1. **15.** 80 cm. **17.** 110·5 cm.; 55·25 cm. **18.** (*a*) 100 cm., (*b*) 136 cm.

CHAPTER XXII

2. 1024 ft. per sec. **3.** 2·92 ft. **4.** 1 ft. **7.** 280. **9.** 254.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	9	13	17	21	26	30	34	38
											4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	12	15	19	23	27	31	35
											4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	11	14	18	21	25	28	32
											3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	7	10	13	16	20	23	26	30
											3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	28
											3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	9	11	14	17	20	23	26
											3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	14	16	19	22	24
											3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3	5	8	10	13	15	18	20	23
											2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
											2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
											2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7529	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	3	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	3716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Angle.		Chord	Sine	Tangent	Co-tangent	Cosine			
Degrees.	Radians								
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0348	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4452	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0023	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·625	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·7081	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6974	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine	Co-tangent	Tangent	Sine	Chord	Radians	Degrees
								Angle.	

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